ACTA UNIVERSITATIS CAROLINAE,
KINANTHROPOLOGICA Vol. 47, 2 – 2011 Pag. 102–106

CHARLES UNIVERSITY IN PRAGUE,
FACULTY OF PHYSICAL EDUCATION AND SPORT[1]
NA HOMOLCE HOSPITAL, PRAGUE CZECH REPUBLIC[2]

EVALUATION OF THE POWER PRODUCED BY MUSCLES

ANTONÍN HAVRÁNEK[1], MARTIN MAYER[1, 2], KAREL JELEN[1]

SUMMARY

Rheological Maxwell-type model is used to obtain a relation between energy input to a muscle and the work produced by it. The key role in the relation plays the muscle activity $1/\eta$ which describes how much energy is converted to work in unit volume element at unit time. The activity is the muscle material value describing how much actin-myosin pairs are engaged in a given place to maintain the work production of the whole muscle system. The modelling of the system has been done and it is now possible to calculate the energy input in special muscle system activities by mechanical means.

Key words: biomechanics, viscoelasticity, energy production, actin-myosin pairs, rheology

INTRODUCTION

There are many processes in which we feel that some energy is consumed (we are working) and crude mechanical models say that we are doing nothing. Typical examples of such a performance is carrying a heavy suitcase via a long horizontal road to a railway station or holding the beer-glass in the stretched arm (see Figure 1). The problem is that even for holding the suitcase or the beer-glass energy transfer to the muscles is necessary to prevent the fall of the items. The way how to express in exact mechanical terms the energy input to the muscles will be given.

METHODS

The main step in solving the problems where we feel that some energy is consumed – some work is done – and the simply calculated macro-work W is zero, is to introduce a viscoelastic model of muscle rheology. Viscous parts of the viscoelastic models are

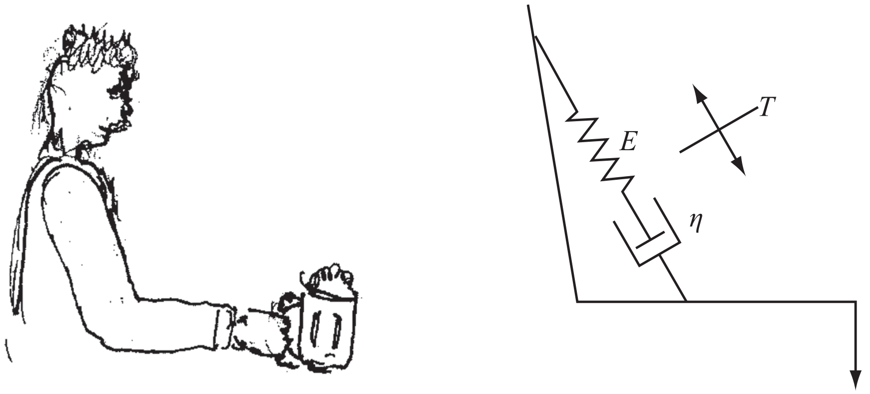

Figure 1. A man holding the glass of beer and the Maxwell model used for rheology descriptions of the act.

combined with dissipation of energy in classical materials. In biomaterials also the opposite sign of energy occurs; by chemical processes the energy input to muscles, the feeding of them, is possible. We have used Maxwell model in the model transcription of the act of holding the beer-glass (right hand side of Figure 1). The weight of the beer-glass and the weight of the forearm *F* create the tension force *T* within the modelled muscle. The constitutive equation of the Maxwell model has the form (see e.g. Ferry, 1980; Havránek, 2007)

$$\frac{\mathrm{d}e}{\mathrm{d}t} = \frac{1}{E}\frac{\mathrm{d}\sigma}{\mathrm{d}t} + \frac{\sigma}{\eta} \tag{1}$$

where η is viscosity, E the Young's modulus of elasticity, e strain (deformation), and σ stress (tension). If the constant stress σ_0 is applied to the model, the Hook's spring deforms immediately and then the deformation proceeds linearly with time t due to the deformation of the Newton's viscous term. When the viscous flow occurs, the forearm is falling down and the beer starts to flows out of the glass. To avoid this accident the man strengthens the arm and remains in holding the glass in horizontal position. What does this action mean in energetic sense? Energy dissipation is combined with dilatation of the viscous part of the model. If we try to stop the dilatation, we must compensate the energy dissipation combined with it by energy input of the same magnitude; we must feed the muscular system. It is relatively easy to calculate the power output P of the Maxwell model of volume V if tension force T acts on it and the model dilate;

$$P = \frac{1}{\eta}\left(\frac{T}{S}\right)^2 V \tag{2}$$

As we employ the concept of energetic compensation, we use the last expression to calculate the necessary power input within the muscle to hold the beer-glass. This is our idea of calculating the power produced by muscles by mechanical means. For details see Havránek (2009), Havránek (2010).

RESULTS AND DISCUSSION

The result given in eq. (2) will be now transformed to a form more convenient for studying the energy production in various muscle systems. Mathematically it means to replace the inconvenient integral form of eq. (2) by the much more flexible differential form. Instead of calculating the power produced in the pre-scribed parallelepiped of volume *V*, the power density *p* produced in a given point x_i at time *t* is expressed. We have got (detailed derivation in (Havránek, 2009) for the power density the expression:

$$p(t,x_i)=\frac{1}{\eta(t,x_i)}\sigma^2(t,x_i) \tag{3}$$

It gives a simple bridge between the stress *σ* at a given point x_i in a muscle and the power density *p* transferred in the point at time *t*. The bridge forms the viscosity term *η* which is the material term describing the state of the muscle in the given place at the given time. If energy input is assumed, $\eta\ (t,\ x_i)$ says how much power *p* is needed to produce the necessary stress *σ* in a given place x_i at a given time *t* in the muscle. By using expression (3) the energy transfers may be studied respecting the tissues differences in different parts of the muscular system and different sizes and forms of the muscles. In soft parts of a muscle the term *η* is relatively small and energy transfer is large, in sinews *η* is very high and nearly no transfer occurs.

The main advantage of this approach is that the power transfer is expressed by only one material value $\eta\ (x_i,\ t)$ and the local stress (tension) $\sigma\ (x_i,\ t)$. We write purposely material value *η* and not material constant *η* as the value of *η* is a function of many variables. It depends not only on the position in the muscle and time but also on the type of a muscle, on the state and condition of the human or animal body and may be on other conditions.

As the extent of power transfer is inversely proportional to viscosity *η*, we shall use for describing the state of the muscle the inverse value $1/\eta$ and we shall call it muscular activity. So the material term called muscular activity increases with increasing power transfer in the given place of a muscle what is a more natural description, The muscular activity increases with increasing density of actin myosin pairs in the place we determine it (see e.g. Fung (1993), Maršík et al. (1993), Nigg et al. (1994) The similarity of myosin motion in actin environment to that of the piston motion in the cylinder of viscous part of Maxwell model is remarkable. The necessary energy input of the process is gained from chemical energy obtained by hydrolysis of adenosine triphosphate (ATP) to adenosine diphosphate (ADP) and so the muscular activity should be correlated with chemical energy calculated for the ATP → ADP process.

By using expression (3) the energy transfers may be studied respecting the tissues differences in different parts of the muscular systems and different sizes and forms of the muscles. If we take into account that *p* is the power density, it comes out from eq. (3) that the power input *P* to the whole muscle may be written as an integral over the volume *V* of the muscle;

$$P(t)=\int_V \frac{1}{\eta(x_i,t)}\sigma^2(x_i,t)\,dV \tag{4}$$

The eq. (4) looks simple but to evaluate it at a given time t the distributions of the variables $\sigma(x_i,t)$ and $\eta(x_i,t)$ in the muscle and the muscle dimensions must be known. It may be a serious task. Nevertheless, some qualitative results may be given without detailed calculations. The power P according to eq. (4) increases with increasing volume V if the muscular activity field $1/\eta(x_i, t)$ doesn't change substantially. So a man who wants to enlarge his output power P must enlarges the volume V of his muscles. The increase of the volume V is achieved by physical training which perhaps may also increase the activity value $1/\eta$ but probably not substantially.

In Havránek (2010) we have in a very crude approximation solved eq. (4) for the case of shot putting. We have got an order of magnitude estimate of the average muscular activity in this case:

$$1/\eta \approx 10^{-4}\ 1/\mathrm{Pa}\cdot\mathrm{s} \tag{5}$$

If we assume that the muscular activity in the case of a man holding the beer-glass (see Figure 1) is lower, say $1/\eta = 10^{-5}\,(\mathrm{Pa}\cdot\mathrm{s})^{-1}$, we obtain from eq. (2) for the power input into the muscle modelled by the Maxwell model

$$P = \frac{1}{\eta}\sigma_0^2 V = 10^{-5}(\mathrm{Pa}\cdot\mathrm{s})^{-1}\cdot\left(\frac{500}{10^{-2}}\right)\mathrm{Pa}^2\cdot 10^{-3}\,\mathrm{m}^3 = 25\mathrm{W}$$

This estimate is obtained if it is assumed that $T = 10\ F$, volume $V = 10^{-3}$ m, $F = 50$ N and the cross-section of the piston modelling the muscle $S = 10^{-2}\ \mathrm{m}^2$. The maximum activity of the muscle decreases with time (tiredness increases) and when it goes beneath the activity necessary to hold the load, we must drop the load.

CONCLUSIONS

The way how to calculate the energy input to a muscle system necessary to produce some work using only mechanical tools has been done. Basic examples, how to use the technique for practical purposes, were given. The procedure is applicable also to situations where no macroscopic work is done as is, e.g., holding a load or walking via a horizontal road.

ACKNOWLEDGEMENTS

This work was supported in part by the Ministry of Education, Czech Republic; Research Plan SVV-2011-263601; GAČR P 407/10/1624 grant. We appreciate this help.

REFERENCES

FERRY, J. D. (1980). *Viscoelastic Properties of Polymers*. Wiley, New York.
FUNG, Y. (1993). *Biomechanics – Mechanical Properties of Living Tissues.* Springer, New York.
HAVRÁNEK, A. (2007). *Úvod do bioreologie* (*Introduction to biorheology*) Karolinum, Praha.
HAVRÁNEK, A. (2009). *Bulletin of Applied Mech*, 5(17), 1–5.

HAVRÁNEK A. (2010). *Calculation of the energy produced in muscles.* In *Human Bimechanics 2010*, *Sychrov*. pp. 356–361. Technical University of Liberec, Liberec.
MARŠÍK, F., DVOŘÁK, I. (1998). *Biotermodynamika (Biothermodynamics).* Academia, Praha.
NIGG, B. M., HERZOG, W. (1994). *Biomechanics of the Musculo-Sceletal System*, Wiley, New York.

HODNOCENÍ VÝKONU PRODUKOVANÉHO SVALY

ANTONÍN HAVRÁNEK, MARTIN MAYER, KAREL JELEN

SOUHRN

Reologický Maxwellův model byl užit k získání vztahu mezi energii dodanou svalu a prací, kterou sval koná. Ústřední roli v tomto vztahu hraje svalová aktivita $1/\eta$, která popisuje kolik energie se změní na práci v jednotkovém objemu za jednotku času. Svalová aktivita je materiálová veličina, která popisuje kolik aktin-myosinových párů je zapojeno v daném místě a daném okamžiku pro zajištění požadované činnosti celého svalového systému. Provedené modelování svalového systému umožňuje vypočítat výkon svalu při zadaném typu pohybu pouze mechanickou analýzou děje.

Klíčová slova: biomechanika, viskoelasticita, tvorba energie, aktino-myozinové dvojice, reologie

Antonín Havránek
antonin.havranek@mff.cuni.cz