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MODEL THEORY AND FOUNDATIONS OF LOGIC

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ABSTRACT
Despite its popularity, model theory based on Tarski's insights is in need of deeper philosophical reflection. A wide range of stances towards it was proposed, some seeing it as project based on fundamental misconceptions, some asserting it reveals the very essence of logic. I would like to balance these extreme views. Of particular importance will be its connection to the problem of logical constants. Identifying logical constatnts enables us to identify logical forms of statements and thus brings us close to demarcating logic. We will see that solving this issue in ways suggested by model theory has its considerable costs, while the gains are rather modest.
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It would not be but a pure folly to doubt that the Tarskian semantics and model theory is a discipline of great importance which contributed significantly to the development of logic and mathematics. Thus it is only natural that it belongs among the most important parts of introductory lectures on formal logic and this should not change. So far, so good. Yet powerful as it is, it remains unclear how we should see it, how to interpret it philosophically. What is actually its place in logic? Is it the core of logic and other ways how to approach it, most prominently from the perspective of proof-theory, belong to logic only derivatevely? Or is the other way round? Or are they perhaps on a par, as far as their logicality is concerned?

This question would be, of course, uninteresting had there not been significantly different outcomes in logic and in philosophy of logic which depend on which answer to it we prefer. The topic of this article will be one of these answers, which depends exactly on seeing the model theory as essential for logic and which was taken by some significant figures of this discipline, including most prominently Tarski himself (in his particular case this was a later turn in his thought, but I will have more to say about this later). The idea is that in the model theory we can precisely specify what makes something a part of logic and thus we can delineate this discipline, ensuring that we will let neither too much, nor too little in. And although Tarskian models are essential to the semantics of classical first-order logic, these model-theoretic demarcations typically have it that logic is actually a much broader discipline. To be sure, taking model theory as the core of logic has got many further consequences, yet it is mainly on those regarding demarcation of the discipline I will focus on.

We will have to distinguish our main topic from a different, albeit related and actually more general one. The general topic which we will touch, as well, is the adequacy of model theory for the study of logic in general. The main point of reference in this regard is Etchemendy (1990). Now, if we reject the radical criticism towards the model theory presented in this Etchemendy's book, we will have settled at least that the model theory can be indeed of some use in logic. Such a situation will naturally call for a closer specification of what we can indeed use it for. And the bold thesis which Tarski presented later, is that in the model-theoretical framework we can say what the bounds of logic are, i.e. demarcate it as a specific discipline and thus, among perhaps other things, show something important about its relationship with mathematics.

We will see that there is both a debate about this kind of demarcations (which I will from now on call The Tarskian demarcation) in general, as well as an internal debate about how it should be exactly spelled out. I will present the gist of both these debates and take a stance towards them.

1. The Tarskian semantics

Let us now begin with recalling what the shape of Tarskian semantics is and see what its main virtues are. Locus classicus of his approach is the 1936 article Tarski (1936) in which he claims the inadequacy of merely proof-theoretic approach to logic. Among other things, Gödel's incompleteness theorems show that it is bound to undergenerate, i.e. to fail to display the relation of consequence in its completeness. Thus a quite different approach is required, one which pays more attention to the meanings of the expressions used in inferences. In other words, a (more)[1] semantic approach. And one which is concerned with what these expressions stand for.

I pressupose that the reader is familiar with standard semantics of classical first-order logic, that is the predicate logic with existential and general quantifier. This antecedent knowledge should serve as a common ground, even if I redescribe it in potentially controversial manners. First of all, Tarski wants to generalize a substitutional approach, which was heralded already by Bolzano. According to a perhaps somewhat anachronic interpretation of Bolzano – and according to Tarski as well – we have to identify a group of elements of our language as *logical constants*, that is as a specifically logical part of our language.[2] It is a separate issue, which elements these should exactly be and we will come to this peculiar problem later. But let us suppose, at least for the sake of this exposition, that it is the standard connectives (conditional, conjunction, disjunction and negation) and the two classical quantifiers.

Now we have to define the notion of a correct substitution, as it is essential for Bolzano's and for Tarski's approach. Here again, I rely on reader's knowledge of this notion. The idea is that in a given argument or an inference, we consider all its substitutional variants, where the variant is defined by mostly obvious restrictions, such as that only a predicate

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1 Although there were such tendencies among the logical positivists, we do not have to say that proof-theoretical approaches abstract from meanings completely. This is not so from the standpoint of inferentialism and also the later development of proof-theoretic semantics shows that the positivist views were somewhat hasty.

2 It is anachronic because Bolzano spoke rather of extralinguistic *ideas*.

can be substituted for a predicate, an individual term for an individual term and so on. Not that there is no room for significant differences in opinions about what counts as a correct substitution, let us remind ourselves that for example Carnap in Carnap (1931) instisted that rules for substitution should be much more restrictive, such as to prevent the substitution of *prime number* for *emperor* in *Julius Ceasar was an emperor*. Obviously enough, such restriction would be difficult to formulate in any systematic way. But there are viable options of more fine-grained substitution rules than those for classical first-order logic which we deal with right now. But this is a seperate issue. Let us pressupose the notion of substitution operative in classical first-order logic.

The notion which we are after in logic is, of course, that of a valid argument. So, according to the substitutional account, an argument from a set of sentences to a sentece is valid if and only if all for all its legitimate substitutional variants it holds that either one of the premises is false or the conclusion is true. A legitimate substitutional variant of a given sentence – and then of a given a argument – is one, in which we substitute only for the expression which are not logical constants (so far, once again, we countenance the classical two quantifiers and the truth-functional connectives) and we substitute only according to the settled restrictions.

It is well known that this approach is problematic because it makes the inference dependent on the language we employ. When the language is not large enough, i.e. if it does not have a large enough vocabulary, it may well happen that we do not have enough substitutional variants of some plausibly invalid arguments to be able to declare them as actually invalid. Obvious examples can be found which are not much weakened even by the vagueness of the notion of intuitively valid/invalid arguments.

Tarski's approach seeks to circumvent this excessive dependence on language. John Etchemendy brings a highly controversial and, as we will see, not completely fair portrayal of the Tarskian endeavour. He describes Tarski's attempt at improving the substitutional approach as proceeding by binding the relation of logical entailment not just to a given actual language but to all possible languages. Even this is inaccurate and leads to Etchemendy's overall inaccurate interpretation, but it is useful heuristically. Thus Etchemendy takes Tarski as trying to step outside the actual language by considering its relation to the world. We do not consider as much substitutional variants of a given argument, but rather we imagine that the word–world relations might change. In our example, individual terms might refer to different objects than they do and predicates may refer to different relations, where relation is understood extensionally. This leads to the familiar notion of a model or structure, the reader's acquaintance with which I pressupose.

So much now for introducing the Tarskian semantics. I would like to note that there is an interesting discussion about whether this standard form it received is what Tarski actually intended. There are authors who think that he did not countenance a plurality of models as we do today, but rather just one univesal model. This would make his position generally much more vulnerable. But the most serious forms of criticism of Tarskian semantics are aimed at the nowadays usual form and thus we can afford to put this historical issue aside.

2. Pressupositions of Tarkian semantics and varieties of critique

Despite the fact that Tarski's analysis has become a part of standard logical curriculum, there has been serious criticism, which nevertheless always respected the achievements of the model theory. Given its proven usefulness, the issue is not whether or not we should somehow accept and use it but rather what to make of its use, how to interpret it. I will like to contribute to the view that the model-theory should be given maybe a little more modest interpretation than usual. But first, let us see the critique of it which is perhaps most direct and most prominent. The one which is due to John Etchemendy.

2.1 Etchemendy – exposition

Etchemendy claims that in logical semantics we have to choose between two basic alternatives, namely doing the interpretational or the representational semantics. Tarski is then supposed to be doing the first one. Let us explain these terms briefly.

In the interpretational semantics the models do not model the ways the world might be but only how the linguistic items might relate to it. Thus a singular term such as the president of the Czech Republic might be interpreted differently in different models, not because a different person can in fact have that political function, but merely because it can mean e.g. what the term *the highest mountain in the world* means in our actual language. The world is thus taken as it is. In the representational semantics, we model rather the ways the world itself might be, thus differing in the reference of the aforementioned individual term simply because different individuals (perhaps more than one at a time or none at all) may be the president. Now, it is obvious that both approaches are dependent on taking some vocabulary as logical, i.e. such that we do not consider either different interpretations of it or the ways the world might make it refer to something else.[3]

Now, according to Etchemendy, Tarski adheres to the interpretational semantics. This claim was much disputed by various authors, but Etchemendy claims that it is indeed the very core of Tarski's approach and his criticism thus cannot be seen as merely a historically interesting thesis about potential confusion of Tarski himself but has to be regarded as an attack on the whole tradition which it produced. Let us begin with marking Etchemendy's basic objections.

On the one hand, the interpretational semantics, even if it would give extesionally acceptable results, i.e. declare as valid all and nothing but the logically valid arguments as logically valid, it would, according to Etchemendy, succeed only by chance, as it reduces the logical validity of some sentences to the material validity of some other ones. Thus we claim that the argument

$$\frac{\text{John is a man and John is single}}{\text{John is single}}$$

[3] It may be good to note right now that it is not reasonable to see the sematics of classical logic as either represenational or interpretational but rather as both at the same time. But we will get to this later.

is logically valid because of the merely material validity (that is the truth of the conclusion or falsity of one of the premises) of all the instances of the following scheme

$$\frac{\phi \wedge \psi}{\psi}$$

or perhaps of

$$\frac{P(j) \wedge S(j)}{S(j)}$$

If we take just the $\wedge$ as logical, then this is plausibly materially valid for all the arguments of (one of the) indicated forms. But that gives us no assurance that the universal claim about all the arguments of this form is not true only accidentally. Etchemendy invites us to consider an argument of the form

$$\frac{\text{John was the president of the USA}}{\text{John was a man}}$$

Now if we allow just the individual term John to vary in the interpretation, then we will still get a an argument such that all its variants are materially valid. And this makes the argument logically valid, despite the fact that it obviously should not be declared as such. It has no materially invalid variants only by chance, so to say. Yet the Tarskian approach cannot distinguish it from the previous, presumably logically valid, one. Or so Etchemendy claims (it immediately comes to mind that the problem here lies in not letting the right expression vary in the interpretation but let us delay this a little bit yet).

If we, just for simplicity's sake, now shift the focus on logically true sentences instead of logically valid arguments, it is obvious that universally quantified sentence's being true does not guarantee the logical (or necessary, a priori or perhaps formal) truth of its instances, but only that they are simply true.

Etchemendy calls this alleged Tarski's step *Tarski's fallacy*. Tarski wanted to reduce the complicated and unclear notion of logical consequence, which was traditionally seen as involved in the difficult epistemological issues, by proposing a relatively clear-cut technical criterion. But his attempt is ultimately fallacious.

I have to say that I am a little bit puzzled by the most basic suppositions of Etchemendy's attack. It appears that the problem is supposed to be hidden in the fact the logical truth of one sentence, say A, is founded in the plain truth of all its variants and therefore in the plain truth of the general statements about the variants (i.e. *all the interpretational variant of A are true*), not latter's being logically true, as well. But I think that any special epistemological status of any claim can potentially be formulated by another sentence which itself is in fact just true. This explaining sentence has to be formulated in some metalanguage which is stronger with regard to the targeted epistemological features of claims. Or should we perhaps demand that the general sentence be true logically and not just simply true in the metalanguage? I am not sure whether such a demand makes sense, just as it hardly makes sense to speak of "trying to try" or "believing that I believe" (as is nicely shown in Brandom (1994)). Maybe the very question whether the metasentence claiming that the original sentence has got only true interpretational variants is true logically or just materially does not really make a good sense at all. And it was not an issue in the first place, as we were examining the logical status of just the original one.

Nevertheless, Etchemendy claims that the interpretational semantics gets the extension wrong, which I think would be, should it be indeed the case, a good reason for refusing it. But let us see some of the alleged instances of extensional inadequacy Etchemendy has in mind. Before that, it should be mentioned that it is in general far from clear what the talk of extension of the logical consequence relation being right or wrong is supposed to mean. I suspect it is simply too naive to suppose that our intuitions hide a totally clear-cut set of logically valid arguments and logically true sentences and that there are no border exemplars, which we ultimately have to simply choose whether to declare or not as logical.

Now for the examples of overgeneration of interpretational semantics. Let us say that there are at least two things (whatever that means) which can be denoted by the individual terms. This means that the following formula(and by applying this analysis, also the sentences which are its equivalents in natural languages)

$$\exists x \exists y((x \neq y))$$

will be declared as logically true. Now, such a sentence can apparently be true only by accident and therefore it makes no sense to declare it as a logical truth[4]. The way the Tarskian analysis escapes having to make this false decision is by varying the domain we quantify over, but that amounts, in Etchemendy's view, to changing the meaning of the existential quantifier, which compromises the original claim that it will be treated as a logical constant. Etchemendy brings more examples of overgeneration, but they are mostly of the same spirit, so we can confine our attention to this simple one.

Etchemendy thus has to interpret Tarski as saying that there must be just one universe of discourse, it must be somehow given what there is in the most general sense. Even if there might be some indications that Tarski might have wanted to head in this direction, it is hardly understandable how such a position can be supposed to be held by the proponents of the Tarskian semantics. Indeed, if his attack is not supposed not to be directed merely at Tarski and thus not to be of mainly historical interest (and I have already noted that there are debates regarding what Tarski originally had in mind), it is hard to see who it is supposed to be aimed at. Nobody is against using different models and it does not make much sense to see them as submodels of one great supermodel (which might perhaps lead to the charge of changing the meaning of the existential quantifier, as Etchemendy formulates it). When adopting a model, there is in principle no claim that the members of the domain have to be existent in the sense of being real. We are free to choose a domain containing pegases and treat it as an intended model of our theory of imaginery creatures. And by doing so, we do not anyhow claim their existence, just suppose it, thus modelling some contexts of argumentation.

[4] It should rememebered, though, that some authors would not oppose such a verdict. Frege though that logic guarantees existence of objects, such as natural numbers.

A problem arises though, namely how many models are we supposed to be using. The problem of possible overgeneration of logical consequences is that we might not have enough models. And it depends on the set theory we use as a background which models there are. This seems to be first of all a problem of indeterminacy, because we cannot say which class of models is somehow the right one. There are more ways to react to this. One of them is, I believe, to regard this as reflection of some genuine vagueness of the notions which are being formalized and thus rendered more precise. This apologetic stance might not appeal to every one, but it is at least not obviously wrong and further discussion is needed here. I find Etchemendy's claim that the axiom of infinity is ad hoc as worthy of attention, though. Be it as it may, it certainly helps to get more plausible verdicts about the logical entailment relation and about the logical truth of statements (thus no statement of the form *there are at most n objects* for a finite *n* will be declared as logically true[5]). So the most counterexmples Etchemendy considers do not arise. Or better, they do not arise in the case of classical first-order logic, which we were considering so far. This is, to remind us, not just because of the happy choice of the underlying set theory, but also because of the particular choice of the logical vocabulary.

Etchemendy follows Tarski in believing that in case of first order logic we have the problem of undergeneration. The invalidity of the ω-rule is claimed to be something in need of a remedy both by Tarski in 1936 and Etchemendy. And thanks to Gödel's incompleteness theorems there is no way we can hope to solve the problem at the level of the classical logic. Yet here we can dispute whether it is a genuine example of undergeneration. Tarski and Etchemendy thus in fact seem to favor a logic which will validate the logicist thesis that arithmetic is a part of logic, as it was presented in Frege (1884). But as interesting as this project was, it is not clear that its fulfillment is so desirable. It is still possible to stick with the more traditional Kantian view that mathematics and logic are indeed separate disciplines. The omega-rule is thus an argument whose validity is not purely logical, but involves our mathematical faculties as well. We have to use the pure intuition to *see* its validity. Depending on one's philosophical background, one can see the (in-)validity of the ω-rule either as an asset or as a problem for a given logic.

Be it as it may with the problem of undergeneration, the overgeneration appears to be much more of a threat. But only so if we interpret the model-theoretic semantic in the interpretational manner suggested by Etchemendy. And that interpretation is rather a straw man for him to attack, as I will try to show.

3. Representational semantics

Another possibility to interpret the Tarskian semantic is, according to Etchemendy, to interpret it representationally. We have already seen a sketch of what that would involve. A given argument is declared valid in case it remains valid under any changes in the world. Described in this way, as it is described by Etchemendy in his 1990 book, it means

5 To make sense of Etchemendy's claim, we cannot say that the axiom of infinity prevents such statements from being logically true by enabling infinite models, as they would not be true even if we had just finite models, though of unbounded finite cardinality. Etchemendy pressuposes the whole time that these finite models must be taken from one big universal model – the world – which then has to be infinite.

merging logic with some sort of general metaphysics. Clearly this does not look like a promising explanatory strategy, as it would involve claryfying the logical notions by means of perhaps even more obscure methaphysical ones. As difficult as it might be to decide about the logical validity of arguments, should it be determined by these criteria, we can see that this would most likely mean running the logical validity together with the analytical one. The inference from

John is a bachelor

to

John is an unmarried man

obviously has to remain valid, no matter how the world changes, as far as the changes do not involve our language (whatever that means). But I think that since at least the appearence of Quine's *Two dogmas of empiricism* we should beware of such a construct. Does Etchemendy's distinction between the interpretational and representational semantic make really sense? And can we accept taking all the analytical entailments as logical ones? I suspect that it was one of the tasks of logic to distinguish precisely between analytically and purely logically valid entailments.

Yet Etchemendy eventually refrains then from this concept of representational semantics and uses the term differently in Etchemendy (2008), partly perhaps as a reaction to criticism, which was issued by Gila Sher in her article Sher (1996). There she accuses Etchemendy of presenting us with a false dilemma, having to choose between the two basic kinds of semantics. Indeed, I think most people acquainted with Tarskian semantics will say, when forced to decide whether it is interpretational or representational, that it is somewhere in between[6]. Indeed, sentence such as

Every bachelor is unmarried

is not declared as a logical truth, perhaps mostly because of the fact that the actual language could have worked differently, many other senteces are not declared logical truths rather because of the way the world could have been, but we cannot in general allot the responsibility just to the language or to the world.

It should be noted that Etchemendy refuses the Quinean attack at the synthetic/analytic distinction, claiming that the attack is based on too narrow a conception of logic. And here we come to the meaning Etchemendy later gives to the represantional semantics[7]. Under this new description he actually endorses it. Logicians, according to this view, always study the inferential properties only of certain expressions, for example the classical connectives and the two classical quantifiers and consider the situations when

6 As should be clear already, we are putting aside the disputes about Tarski's opionons in the 30's and talking about the model theory in its modern shape. Saying that it is somewhere in between the two approaches is a somewhat simplifying expression of what is better expressed in MacFarlane (2000), namely that various models model different contexts. That is, not necessarily interpretations or states of affairs.

7 Though it is perhaps a little dubious why he calls it so. His exposition can be found in Etchemendy (2008).

the members of the other parts of the vocabulary, such as *bachelor* or *unmarried man* change their denotation. And herewith we come to the problem of logical constants.

4. Logical constants

Logical constants can be characterized as the elements of language which determine the logical properties of sentences, the only ones, which, as Quine puts it in Quine (1986), occur in logical truths or logical entailment relations essentially. The problem is that it is not clear which elements of the language should be counted as logical constants. Tarski himself expresses in the 1936 article the opinion that the division between logical and non-logical constants cannot be completely arbitrary, but it might be impossible to demarcate the logical constants quite principally, as well.

Etchemendy thinks that the choice of logical constants is indeed arbitrary, because every element of language has some logical properties and it is only up to us, which collection of linguistic items we want to study from the logical point of view. He calls the problem of finding the right set of logical constants a red herring. Every element of language has got some specific logical properties and it is up to us whether we find it useful to study them. It is thus very well possible to study for example the logic of "color-words", which typically involves inferences such as

$$\frac{\text{This apple is red all over its surface}}{\text{This apple is not green all over its surface}}$$

Or inferences such as

$$\frac{\text{This apple is red}}{\text{This apple is coloured}}$$

The traditional logical constants were historically given special treatment only because their logical properties are particularly important or particularly amenable for logical analysis. Now, this approach is of course a possible one, but it obviously makes the very notion of logic very vague. Or rather very broad. Logic is thus transferred into a general study of inference. It is thus important that Etchemendy does not regard inferences such as

$$\frac{\text{Socrates is a man}}{\text{Socrates is mortal}}$$

as an enthymeme. This brings him close to positions of Robert Brandom. But what drives him far away from Brandom's position is that he does not endorse logical expressivism, which is a corollary of the fact that he does not think that logic has got a specific vocabulary. He probably also does not agree with Brandom's identification of meaning of an expression with its inferential properties. Meaning can hardly be, according to Etchemendy's picture, constituted by a position a given sentence – and derivatively also its constituents – have in the overall inferential web. Without logical expressivism it is mysterious how inferentialism could work. Thus his view of meaning seems to be irreducibly representationalist. And such a view has faced many problems in the recent decades of philosophy of meaning.

Keeping that aside for now, we should say that there are ways how to characterize logical constants in the Tarskian framework, ways which are vulnerable to criticism but are not completely arbitrary, thus at least partly fulfilling Tarski's original desideratum. The core of the proposal comes from Tarski himself, from his lecture, given well after his original articles about semantics, namely Tarski (1986). In it he generalizes the Klein's Erlangen programme of demarcating notions of various geometries.

5. Invariance criterion

The key notion in this Tarskian enterprise of demarcating logical constants is that of invariance. For example, the notions of Euclidian geometry, such as being an isosceles, are invariant under permutations of the universe of points which preserve similarity. A permutation of the points naturally induces a permutation of the sets of points, of sets of sets of points and so forth (the permutations on higher levels). Thus a similarity-permutation is one which maps a given triangle onto another triangle, which can be proportionally smaller but remains an isosceles if and only if the original one was such and so forth.[8]

Now the first attempt to define logical notion is to say that they are the ones which are invariant under all the permutations. The argument starts off with the premise that logical notions should be the most general ones. Now, when we relax our demands on the class of permutations, under which the notions of a given discipline are supposed to be invariant, we get an increasingly general discipline. Logic therefore goes as far as is possible in this setting. Here it actually seems that Tarski is speaking about a single universe (the "world"), which might give support to the earlier mentioned Etchemendy's interpretation of his endeavour. Yet this approach needs to be ammended, as it would allow for example the quantifier $\exists_{\forall}$, which would behave as an existential quantifier in case there are some cats and as the universal one otherwise. It would be thus indeed invariant under all permutations, but it indeed feels strange to accept it as a logical notion.

This problem was nevertheless fixed, as later authors, such as Sher, began to consider not just all the permutations of a given domain, but rather bijections between various domains. Sher calls this typically an isomorphism, but she does not mean that it respects the interpretation of non-logical symbols in a given model, that is preserves the properties of the members of the universe, rather it just preserves the properties of higher level-objects (sets of objects, sets of sets and so on), i.e. those which are induced from the original domain of the given structure. Let us see which notions thus get counted as logical.

To begin with, no individual constant passes the test. If we have e.g. the constant 0 in the language of arithmetics, it can be mapped e.g. to 1 even in the same structure of natural numbers, that is in the standard model of Peano arithemetic. Of the first-level

[8] A more systematic and less hasty exposition can be found in Tarski (1986).

predicates (or sets) we get counted the universal relation and the empty relation, from the first-level binary relations identity, the non-identity (that is the complement relation to that of identity) and so forth.

When we start talking about the quantifiers, understood as second-order predicates (the predicates of predicates, or sets of sets), the list gets significantly extended. First of all the two classical quantifiers, that is the existential and the universal one clearly pass the test. A non-empty set get clearly mapped on a non-empty set by any bijection between two structures, as well as an empty one. The same consideration holds for the universal set. But we can go futher and consider any quantifiers regarding the cardinality. Thus any quantifier demanding that a set of objects satysfying a given formula has got a certain cardinality is declared as logical. Just for illustration consider

$$\aleph_1 x\phi(x)$$

But we can generalize even more. The traditional quantifiers are, from our point of view, sencond-level unary predicates. But we can consider also second-order predicates of higher arity, for example the relation *most*(thus being able to formalize such propositions as *Most A's are B's.*). And we can also consider unary second-order predicates, which are applied to first-order relations of higher arity than one, say the binary ones. Or we can have hybrid relations, which are applied for example to an individual and a predicate, such as the relation of membership, understood not as a relation between elements of the universe but between the elements of the universe and sets of elements thereof.

Gila Sher in her Sher (1991) presents the results of this approach in a very comprehensive manner. The book is thus reccomendable for those who want to get a more exact idea of the results of this demarcation. Yet we have now seen what might at least give the basic flavor of what we get. Now, can we be happy with such a result? When it comes to the problem of extensional adequacy, it is clear that overgeneration is much more of a danger than undergeneration in this case. It can even be shown that any structure can be characterized by the means of the bijection-invariant operators, among other the standard model of Peano arithmetic, see Bonnay (2008). What are we to make of this? There are authors who see this as a mark of adequacy of this demarcation, such as Sher, and also ones who see it also a clear mark of problem, such as Dennis Bonnay.

5.1 Virtues of the demarcation

This approach gives a demarcation which is very precise and systematic. From a certain point of view, given by Sher, the classical logic, confined to its two quantifiers, appears to contain a relatively arbitrarily small fragment of what the whole system, which she calls *universal logic* has to offer. For example the cardinality-quantifiers are very similar to the two classical ones from the set-theoretic point of view. The system she countenances is actually still a first-order logic, as it does not contain the second-order quantifiers, it might be called the generalized first-order logic[9]. Or rather generalized

[9] Of course, this demarcation based on invariance can be exted to higher-orders and Tarski originally does exactly this. Yet Sher shows that it is actually quite enough to consider just the first-order generalized quantifiers.

first-order logics, as we may choose to work in smaller systems, such as the classical logic, or the classical logic enhanced just by the quantifier "there are infinitely many" or "there are uncountably many" etc. As we speak in the case of merely propositional logic of the completness of the connectives, i.e. that e.g. the negation together with disjuction are capable of expressing all the boolean functions, so we can speak in a slightly figurative manner also of the completness of the first-order logic. The universal logic which we just sketched can be thus seen as complete with respect to what can be expressed by the means of first-order quantifier-operations defined over structures.

Despite the mentioned similarity between the two cases, the contrast is great, as well. As we content ourselves with just two (or, of course, one) connectives in the case of completness of the propositional logic, in the case of first-order logic we need a host of quantifers which is very difficult to oversee (at least as difficult as to oversee the set theory). Sher claims that although this approach blows logic up to an unprecedented degree and makes it thus immensly complex, it compensates for this fact by being principled, i.e. by being based on a single and clear principle. Informally said, logic is a discipline which abstracts from the identity of objects, all objects are equal for it. This might remind us of Kant's conception of logic. Kant claimed that logic abstracts from the relationship of cognition to its object. This probably cannot be said of the universal logic Sher proposes. This logic treats of a relationship of cognition to objects, though in a very general way, surely not of relationship to any concrete objects.

But even this might be slightly doubtful. Of course, we have to accept the specific understanding of *object*, i.e. the member of domain of some model-theoretic structure and not, for example, a set theoretical construction on these objects. An existential quantifier or any of the generalized ones can be seen as operation on the structures and as such perhaps also as an abstract object. This is not a refutation but it shows that this approach, not much surprisingly, pressuposes that the notion of an object is already settled. It is up to the reader to decide whether logic can be build upon such a presuposition.

Anyway, Sher praises general logic for displaying the form of our reasoning. Logic becomes the discipline of the formal. Certainly we can choose to understand formality as what is captured by this generalized logic. I suspect that any non-mathematical or intuitive notion undergoes some changes when it gets treated mathematically, at least in the sense of being made more precise and thus bereft of its vagueness which might have contributed to its importance and vivacity.

But there is one larger problem. Or perhaps two related ones. The first one might be the concern with possible overgeneration. Again, such an issue typically cannot be decided definitely, as it is not clear with respect to what the given logic is supposed to over- or undergenerate (some sort of "right" relation of logical consequence). Yet in this case we see that a lot of set theory has creeped in. Indeed, the situation recalls the second-order logic and Quine's dictum that it is a *set theory in sheep's clothing*. Perhaps logic should not be able to speak of such things as various infinite cardinalities. For one thing, it might jeopadize a status which is often attributed to it, namely being topic-neutral. I believe that the set theory is a topic and a large one! Logic is thus to contain vocabulary, which is relevant only to one specific discipline, which also seems hardly acceptable. Yet of course, it will depend on broader philosophical stances towards logic, whether one sees this as problematic. It is possible to renounce the topic-neutrality as a desideratum of logic.

The more acute, though related worry is that should this all be logic, then it would somehow lack real foundation. After all, the set theory does not seem to be a safe foundation for lots of reasons. In a way, we do not really understand what the quantifier $\aleph_1$ means, since we do not know whether the continuum hypothesis is true. Actually, since it was shown to be independent of the axioms of ZF in (classical) first order logic, it is even quite reasonable to say that we cannot declare the continuum hypothesis neither as true nor as false. By this I do not mean only that we cannot know whether it is true or false but rather that it itself is neither true nor false. In order to say either, we would have to know what the *real* model of set theory is, which I believe is not the case. We actually have no way of veryfying that there is even any model at all.

Again, similarly to the case of the second-order logic, we can even formulate in the purely logical language of Sher's universal logic a sentence which is equivalent to the continuum hypothesis. Logic thus has to declare CH either as true or false, which is very hard to swallow. Sher tries to defend her system in a similar way in which Shapiro tries to defend the second-order logic, charging its opponents of "foundationalism" in Shapiro (1991). Sher claims in her article Sher (1999) that when we try to explicate logic, it is bound to lose its character of foundation of all knowledge, it has to be made partly dependent of something, which helps to explain it. This much, I believe, is true. Yet of course the question remains how complicated can the tool, e.g. the set theory, which we use to explicate logic be. In general, it is up to us and our preferences, though founding logic on something as complicated as the set-theory seems to be too much. We might be prepared to revise some of our intuitions about logic as a foundation of cognition, but this amounts rather to changing the subject that offering a novel account of logic.

Furthermore, this criterion does not rule out some very dubious quantifiers, because it pays attention, so to speak, only to the quantifier's good behaviour on structures of every cardinality separately. We can thus think of a quantifier, which behaves as an existential one on finite models and as a universal one on the models of infinite cardinality. Furthermore. We can think of quantifiers which are extensionally equivalent to, say, the existential quantifier, but have obviously a different meaning. For example a quantifier, which is – taken as a second-order predicate – true of a set under the conditions that it is non-empty and water is H_2O. Nice overview of these examples of overgeneration can be found in MacFarlane (2009).

Indeed, this criterion is, as John Macfarlane calls it in MacFarlane (2000), actually not semantic, but a presemantic one, as it does not deal with the relationship between the extension of logic operators and linguistic items, by which they are supposed to be denoted. Gila Sher asserts that logic is indeed dealing only with the extensions of our linguistic expressions. Yet it is difficult to see why such a restriction should be reasonable. Indeed, it almost appears as an inversion of the Kantian restriction that logic should not speak about the relationship between cognition and its objects.

6. Final assessment

Does this all mean that Tarski's approach to logic is incorrect or flawed? We have to be prepared to accept that every attempt to demarcate logic is bound to be only partially successful, since the guiding intuitions are too vague and may always produce new objections against individual proposals. Yet we have seen that there are many objections this specific approach has to face. Not that there are no possibilities to defend it or to amend it. It worth mentioning that Dennis Bonnay is, among others, trying to generalize the notion of invariance and speaks of invariance not just under bijections or isomorphism but also under partial isomorphism and so forth. He thus shows that it is possible to reduce a lot of the problematic interconnections of logics based on invariance criteria with what should be rather extra-logical affairs and especially the dependency on problematic set-theoretical assumptions such as the continuum hypothesis. Even the classical first-order logic can be characterized by means of invariance criteria, namely notions invariant under monadic surjective functions Bonnay (2008). These are for themselves very interesting results which give us the new possibilities of understanding the various logical systems and understand what the difference between the classical logic and its Tarskian amplifications – including the second-order logic – ammounts to.

As I mentioned in the beginning, Tarskian semantics surely is a powerful and handy tool for studying various logical systems. Yet it seems hardly acceptable to see it as revealing the essence of logic (perhaps nothing can achieve such a goal). Yet in the case of the first-order logic we have the happy circumstance that it is complete. Or perhaps we should use a different term, such as axiomatisable. As Etchemendy points out in Etchemendy (1990), the term completeness suggest that the model-theory is something more basic and secure, something the axiomatisation is to be tested against. And I see his suggestion to look at things from the opposite perspective as a very healthy one (this idea is developed in Peregrin (2006) and Peregrin (2014)). This means regarding the axiomatisation as something, which can be seen as being certainly in the realm of logic, at least in the sense that the axioms and inferential rules are of themselves plausibly logically valid. Then the model theoretical system of classical first-order logic gets its foundation by the completeness theorem. Yet this cannot be of itself a sufficient argument in favor of some kind of exclusiveness the classical logic. First of all, the notion of plausibility of the deductive system is problematic, invoking the traditional notion of an axiomatic system as a system of self-evident truths, which is hardly tenable, given the many alternative logics. The second problem is that in the Tarskian semantic we can formulate systems which are stronger than the classical logic and still axiomatisable, such as the system of classical logic plus the *there are uncountably many* quantifier. And here the problems with dependency on the set-theory and epistemological ill-foundedness reemerge.

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