Do Student Teachers Attend to Mathematics Specific Phenomena when Observing Mathematics Teaching on Video?[1]

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Abstract: This article describes the results of an investigation into pre-service teachers' ability to notice mathematics specific phenomena in a lesson observed on a video recording. Thirty mathematics education students' written analyses of a viewed lesson were subjected to a selective content analysis. The results of both qualitative and quantitative nature conform with earlier research on pre-service teachers' lesson analyses and, in addition, bring detailed report not only on the participants' ability to notice but also on categories of content-related observable aspects of teaching. The discussion focuses on situating the findings within the framework of pedagogical content knowledge and indicates ways to further link the ability to notice to both teacher development design and effective teaching practice.

Keywords: ability to notice, professional vision, pedagogical content knowledge, lesson analysis, pre-service mathematics teachers, mathematics specific phenomena

1 Introduction

In the present classroom, it does not suffice that a mathematics teacher prepares a lesson well and then enacts it. The role of the teacher is far more complex − pupils should be taking an active part in their learning, with the teacher acting more in the background than traditionally but still guiding their pupils towards the knowledge they are meant to build. Naturally, such ideal lessons cannot be prepared in advance in every detail; rather, the teacher is expected to be able to appropriately react on the spot, to the pupils' suggestions, solving strategies, unexpected situations, etc. The question arises how well teachers and student teachers are prepared for this aspect of their work, how developed their ability to notice relevant aspects of a teaching situation is. While teaching future mathematics teachers at the university, we noticed that our students' written accounts of their teaching practice tended to include general pedagogical comments but rarely comments related to the way they developed their pupils' mathematical knowledge. We have got similar results when student teachers reported on their observation of other teachers' teaching. To focus their attention on features related to mathematics teaching and learning, we asked them to complete a specific task during their teaching practice in which they were to describe three interesting moments from their teaching or from the teaching of others which relate to mathematics learning and teaching. Still, in the last four

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years at least 40% of the students' responses have been related to mathematics teaching and learning very loosely or not at all. Thus, we became interested in the characteristics (such as the quality and the objects) of student teachers' attention.

2 Theoretical Framework

In research, we find several constructs which try to capture the characteristics of attention from different standpoints: noticing, professional vision, knowing-to, attention-dependent knowledge.

According to Sherin and van Es (2005), *noticing* involves (a) identifying what is important in a teaching situation, (b) making connections between specific classroom interactions and the broader concepts and principles of teaching and learning that they represent, (c) using what teachers know about their specific teaching context to reason about a given situation. Another term describing the same phenomenon is *professional vision* which involves "socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (Goodwin, 1994, p. 606). Thus it makes sense to speak about teachers' professional vision which Sherin (2007) describes as consisting of two distinct, but intertwined, sub-processes: *selective attention* and *knowledge based reasoning*. Teaching is a very complex process. There is a lot going on in every mathematics lesson and the teacher cannot pay attention to everything, certain features stand out for him/her – then we speak about selective attention. Once "the teacher's attention is drawn to a particular event, next the teacher will begin to reason about the event based on his or her knowledge and understanding" (Sherin, 2007, p. 385).

The (student) teachers' ability to notice is important for the development of what Mason and Spence (1999) call *knowing-to*: "Knowing-to is active knowledge which is present in the moment when it is required." They distinguish this kind of knowledge from knowing-that, knowing-how, and knowing-why. Knowing-to triggers the other types of knowing and thus its absence blocks "teachers from responding creatively in the moment" (ibid). While Mason and Spence mostly concentrate on the way knowing-to develops in pupils (e.g., while solving problems), they also touch on educating teachers to be able to know-to:

"We propose that knowing-to act in the moment depends on the structure of attention in the moment, depends on what one is aware of. Educating this awareness is most effectively done by labelling experiences in which powers have been exhibited, and developing a rich network of connections and triggers so that actions 'come to mind'." (ibid)

In the same spirit, Ainley and Luntley (2007) propose the term *attention-dependent knowledge* for "highly contextualised propositional knowledge that is made available by attending to aspects of the classroom situation", that is, the knowledge that enables teachers to respond effectively to what happens during the lesson. It can only be revealed in the classroom.

"It is knowledge that becomes available during the complexity of the progress of a lesson, often in response to instances of pupil activity that could not be predicted on the basis of the teacher's subject or pedagogical knowledge. However, we conjecture that it is attention-dependent knowledge that enables teachers to act effectively in response to what happens during the lesson." (ibid)

We consider the teacher's professional vision part of pedagogical content knowledge (PCK) (Shulman, 1986). For example, Bromme (2008) claims that PCK can also be seen in the ways the teacher "takes into account pupils' utterances and their previous knowledge". An (2004) stresses four aspects of the effective teacher's activity in the classroom which are part of PCK: building on students' mathematical ideas, addressing and correcting students' misconceptions, engaging students in mathematics learning, and promoting and supporting students' thinking mathematically. In order for the teacher to take into account the pupil's utterance and build on his/her understanding, he/she has to notice the importance of this utterance in the first place, put it into the appropriate context, interpret it, and only afterwards use it.

There is an agreement in literature that the ability to notice can be studied (and also developed), among others, by letting (student) teachers analyse video recordings of the teaching of others and/or their own (e.g., Borko et al, 2008; Goffree & Oonk, 2001; Hošpesová, Tichá & Macháčková, 2007; Krammer et al, 2006; Lampert & Ball, 1998; Llinares & Valls, 2009; Muñoz-Catalán, Carrillo & Climent, 2007; Santagata, Zannoni & Stigler, 2007; Sherin & van Es, 2005; Star & Strickland, 2008; Tichá & Hošpesová, 2006). Most of the studies confirm that (student) teachers must learn what to notice. For example, Santagata, Zannoni and Stigler (2007) found out that "more hours of observations per se […] do not affect the quality of preservice teachers' analyses", and on the other hand, Star and Strickland (2008) claim that the ability to learn from observations of teaching "(either live or on video) is critically dependent on what is actually noticed (attended to)". Blomberg et al (2011) point out that videos of lessons "represent both subject specific and generic aspects of instruction and thus have the potential to activate knowledge of both these aspects". It is this perspective that our study assumes: distinguishing between mathematics specific and generic aspects of teaching, we will focus on the former of the two.

2.1 Research on noticing mathematics specific phenomena

From research studies on noticing, we will present only those which meet two criteria. First, as we observed that our student teachers' attention was particularly drawn to general pedagogical aspects of teaching, as already stated above, we choose studies which specifically investigated whether and in what way (student) mathematics teachers attended to phenomena specific to mathematics teaching (as opposed to phenomena pertinent to teaching any other subject). Second, only work which aims at the types of phenomena which (student) mathematics teachers noticed in a mathematics lesson without any previous training is presented. As most of such studies are of the intervention type, primarily the (student) teachers' noticing of phenomena related to mathematics prior to the intervention will be presented here.

In their larger scale study, Santagata, Zannoni and Stigler (2007) investigated the quality of the analyses of 140 student teachers divided into two groups. The first group of students was asked to observe a video of a whole mathematics lesson and the second one saw parts of the lesson only, in order to reduce the complexity of the teaching situation and focus the students' attention to fewer phenomena. The results were similar in both cases. One of the categories the authors coded for in students' analyses was Mathematics Content. The comments were coded as high quality if they analysed teacher's and pupils' actions in relation to the mathematical content. The analyses were given a score of 1 for this category if they included mostly low-quality comments; a score of 2 if they included a balance of high- and low-quality comments; and a score of 3 if they included mostly high-quality comments. The results for the pre-test were 1.37 for the first group of students and 1.57 for the second and the authors concluded that in "the pre-test, the comments tended to be about general didactic choices and, when the mathematical content of the lesson was mentioned, only seldom were mathematical ideas used directly to discuss the teacher's actions". The score for the post-test was 2.02, 2.05 respectively, and "participants more often used their knowledge of mathematical concepts to shed light on what they observed, or to argue for the efficacy (or lack thereof) of the teacher's choices".

Similar conclusions were drawn in some small scale studies. Star and Strickland (2008) conducted a study with 28 student mathematics teachers. On the basis of an expert analysis of a mathematics lesson, they created a set of questions (multiple choice, yes/no and short answer questions) which the students were asked to answer from their memory after seeing a lesson on video. They found that the students did not enter mathematics teaching courses with well-developed observational skills and specifically that "preservice teachers were not particularly observant of more substantive features of classrooms, particularly mathematical content" and this remained true even after the intervention course. When content was noticed, the students "tended to comment only about whether the content was presented accurately and clearly and/or to provide a chronological description of what the teacher wrote on the board during the lesson". Star and Strickland conclude that the students "largely did not notice subtleties in the ways that the teacher helped students think about content".

Alsawaie and Alghazo (2010) conducted a small scale study in which 26 student teachers took part. They found out that prior to the video based course, the students' analyses of a mathematics lesson tended to include chronological descriptions of most what happened in the lesson with no interpretation and with no identification of noteworthy events. Similar results were reached in a small scale study by Sherin and van Es (2005). Both studies report a significant change after the course in the choice of noteworthy events and the way the participants saw them – they used fewer evaluation comments and more evidence-based comments.

The above research shows that (student) teachers, without guidance, notice general educational phenomena or phenomena of classroom management rather than phenomena related to mathematics in the video recordings of mathematics teaching. However, they do not go into much detail into what kind of mathematical phenomena were noticed and which ones were neglected. Thus, in our research, we have decided to focus our attention on the mathematics specific phenomena only. This also enabled us to reduce the complexity of coding criteria which some of the authors report (e.g., Santagata, Zannoni & Stigler, 2007) and which we, too, encountered when creating a code system that would encompass all elements of the discourse found in student teachers' written comments on video recorded mathematics lessons.

2.2 Mathematics specific phenomena

None of the studies above includes a unique category for mathematics specific phenomena as we see them. For example, in Star and Strickland's (2008) work, it would span three categories: Tasks, which refer more generally to activities pupils do (and thus include both generic and subject specific aspects of teaching), Mathematical Content, which includes representation of the mathematics involved (graphs, equations, tables, models), examples used, and problems posed, and Communication, which includes pupil-to-pupil as well as teacher-to-pupil communication aspects, such as questions posed, answers or suggestions offered, and word choice. Santagata, Zannoni, and Stigler (2007) code for five dimensions, three of which overlap with mathematics specific content: the Mathematics Content (for coding all the comments – "comments that did not mention the mathematics presented in the lesson were coded as low quality; comments that analysed teacher's and students' actions in relation to the mathematical content were coded as high quality"), Student Learning and Critical Approach dimensions. Finally, Van Es and Sherin (2010) used dimensions to code for each comment, and one of them was Topic, which included Mathematical Thinking, Pedagogy, Climate, Management, or Other. In our study, the mathematics specific category would span Mathematical Thinking, which refers to mathematical ideas and understandings, and the part of Pedagogy which refers to techniques and strategies for teaching the subject matter.

By *mathematics specific phenomena* (or MS), we will mean the phenomena that could be observed, explained, inferred or interpreted in relation to either mathematical or didactical issues pertaining to the teaching or learning of mathematics as opposed to other subjects. Thus, MS category can be seen as a part of professional vision of a teacher of mathematics as opposed to a teacher of other subjects. Further clarification of this concept will be made by examples in the analysis below.

3 Methodology

We focus on the following research questions:

1. Which MS phenomena do student teachers attend to and which do they miss when not given a specific focus area (selective attention)?
2. How do they interpret these phenomena (knowledge-based reasoning)?

The data consists of written analyses based on a video recording of one particular lesson, namely, an Australian mathematics lesson from Grade 8 from TIMSS Video Study 1999. We consider the lesson to be reasonably rich in generic and subject didactic content (Blomberg et al., 2011) and thus it offers a solid base for our study. The topic was the division of a quantity in a given ratio. The teacher starts out introducing the topic, providing pairs of students with wooden blocks and leading the class through a sequence of graded modelling tasks to arrive at the concept of division of a quantity in a given ratio. Following this introductory part, the pupils are asked to create their own problem based on dividing in a ratio and have their partner solve it. Then the teacher draws her pupils' attention to practical real life applications of this type of problem and asks them to create "a story" based on the problem they had made up previously. From the pupils' elicited answers it is evident that the task was not clearly stated and the teacher tries to clarify the situation. Then the teacher quickly shows another way to solve a problem by using fractions and asks pupils which method they prefer. Individual textbook practice assignment follows, and the lesson's last part is spent on the initial stage of an investigative activity with "Smarties" sweets, focused on statistical topics, such as frequency and percentage. Further relevant aspects will be described in the text below.

Thirty students,[2] future mathematics teachers, who enrolled in mathematics education (here ME) course at the authors' department between 2008 and 2011, participated in this study. Twelve of them were students of the first semester-long course in ME (Group A), 13 were students from the second, continuation, course and 5 students from the third course (Group B). All of them had studied mathematics for three years at the university level before taking the ME courses. Their teaching practice takes place after the first course in ME, thus it can be assumed that only Group B students had had between 8 and 16 mathematics lesson observations and between 12 and 24 lessons of their own teaching (to pupils 12 to 19 years old). Both groups had participated in a pedagogical-psychological practicum in which they observed lessons of different subjects in schools for one semester one day a week and had seminars with a psychologist and a general educator speaking about their observations.

In the study, students were asked to freely reflect on the lesson at home; they were instructed to watch the video or parts of it as many times as they needed,

2 We use the term pupils to refer to pupils in the recorded lesson and the term students to refer to university students, future mathematics teachers.

pausing, rewinding and forwarding it at their leisure. No expected length or structure of the text were specified. The students submitted their analyses in Word files or via Moodle.

At the beginning of the series of three ME courses, the students had no experience with analysing videos, however, as the task was set within the ME course, we presumed that they would naturally tend to notice or write about MS aspects of the lesson rather than the generic ones.

Within the ME courses, the first of the authors quite frequently uses analyses of video clips as class activities to illustrate a teacher's approach, pupils' reactions, teacher-pupil communication, etc., or to elicit the students' views of mathematics teaching, stimulate their thinking about teaching a particular concept, etc. However, the development of the ability to notice is not the main focus of the courses.

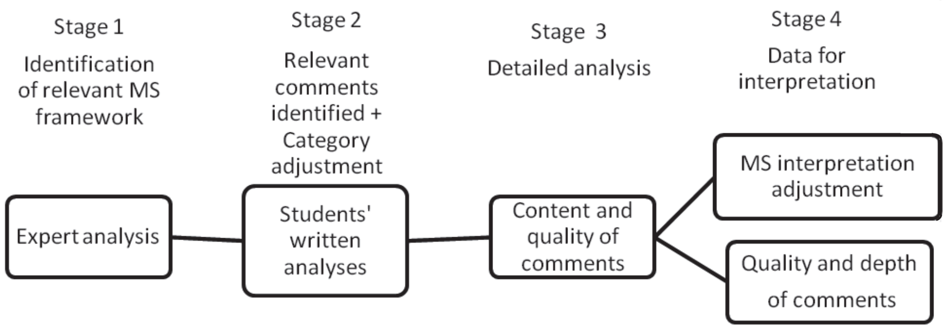

Figure 1 Data analysis. The diagram of the data extraction and organisation

Figure 1 represents the entire process of data extraction and organisation which ended with a theoretical saturation (Strauss & Corbin, 1998). Similarly to Star and Strickland (2008) and Blomberg et al. (2011), we carried out an expert analysis of the mathematics lesson in question in the first stage. An expert analysis framework for this particular video recording was drawn based on multiple viewings and was subject to various revisions of three independent ME experts. The process of this analysis was not meant to simulate the context in which students were writing their analyses (as we assume that the majority of them did not resort to multiple viewings, for example) but rather aimed at identifying as many MS significant phenomena as possible in order to encompass the wide range of potential responses. There were 11 main coding categories identified by and commented on in the expert analysis.

In the second stage, the students' written analyses were coded for the expert analysis categories in the Atlas.ti software (while scanning the text for comments on any additional MS phenomena). There were some minor adjustments made to the categories: the category *Teacher's mathematical error*, which was initially spread across other categories, was added to the framework, and the category *Process vs. concept* was omitted as no data were coded in the students' analyses as relevant.

 After these adjustments, the category system was discussed in a group of researchers so that inter-coder reliability was ensured.

The data extracted by applying the system of the 11 categories would be significant in telling us *whether* and *what* the students noticed and chose to report on. However, in order to get a deeper understanding, we decided to analyse the categories further, hoping that such analysis would lead to answering our *how* question. Thus, in the third stage of the analysis, we identified codes which showed content or depth of individual remarks within each category. Some of these codes were already identified in the expert analysis, some resulted from the content of the students' text. The nature of such sub-division proved to be in some cases disjunctive (e.g., students either interpreted the last activity in the lesson as aligned with the lesson topic or not) and cumulative in others (e.g., a student could report a *specific* case of dealing with a pupil's response as well as comment *in general* on the teacher's use of pupils' feedback from activities at the same time). Sometimes the subcategories were established based on specific interpretation of a situation (e.g., whether students described the ratio-quantity segment of the lesson as teacher explanation or student discovery – an interesting example of contradictory interpretations of an identical situation captured on video).

The main purpose of creating the code system was to get as much relevant insight into the qualitative aspect of the comments as possible. Thus, for example, the subject of elaboration was sidestepped as non-relevant for our study in some cases (e.g., codes in category *Reinforcement of previously learned concepts* offered natural opportunities for simple description, especially as the phenomenon was a marginal one in the lesson) while in others a simple description of a situation in the students' analyses resulted in the loss of MS aspect of the comment and thus was not used as input in our study (e.g., when a student simply states that "in the next stage, the teacher expresses given parts as fractions", the recount itself lacks any MS dimension). In fact, we began to notice that, apparently, not every phenomenon from the framework was being viewed by the students from the MS perspective. In consequence, in the final stage of the analysis, we decided to consider eight codes as not directly concerned with an issue specific to the domain of ME or mathematics. For example, if a student described or even praised the use of blocks in the lesson, *without* referring to their modelling role, we did not assign this comment a MS aspect. In the same way, the set of comments made about the teacher's dealing with pupil responses *in general* (e.g., "The teacher answers her own question instead, leads the pupils to an answer while she should let them formulate their own answer.") while specific cases of pupil response and/or the teacher's reaction with specific mathematical content were coded as MS (e.g., "Pupils were further asked to come up with a story about why we should divide 210 Smarties in the ratio 2 : 5. This caused some difficulties, because they answered that they wanted to divide them by colour or among 7 people."). After much deliberation, we also chose not to include general comments about real-life connections, motivational aspects of the pupil-posing activities and non-elaborated descriptions of the activities. By omitting these non-MS comments, we extracted *data adjusted for MS dimension*.

The detailed analysis also enabled us to code for the use of notions and terminology frequented in the theoretical groundings of the ME courses as well as for the student's propensity to criticize and/or offer alternatives to the teacher's MS related actions.

4 Results

The students' written analyses differed in length, from a single paragraph to two pages (from 76 to 1185 words, with the mean value of 460 words). The ability to notice MS phenomena can be measured, for example, by the number of categories that appeared in individual analyses. None of the students considered the maximum 11 categories, while one commented on 10 categories, and the lowest scoring four students mentioned one category only (each a different one). Fig. 2 divides the analyses into four subgroups. It is noteworthy that 18 students (60%) reported (from the MS perspective) on less than a half of the 11 identified categories.

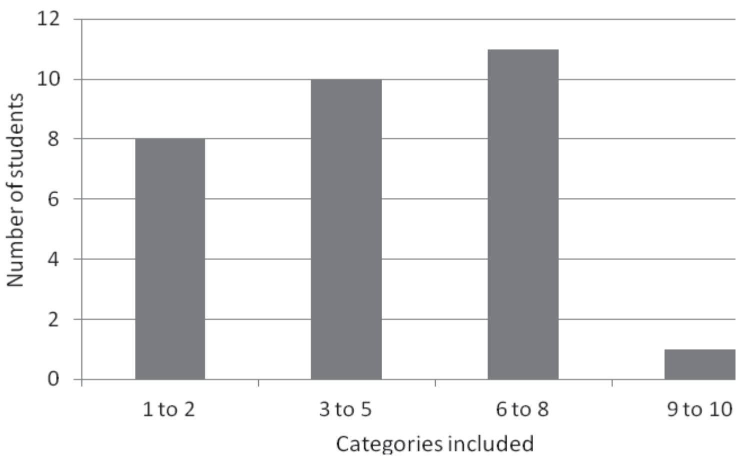

Figure 2 Number of categories noticed in analyses

Figure 3 depicts which particular phenomena were most or least noted by the students. We can see that the *Pupil problem-posing activity* received most attention (albeit of only 63% of total students). It was rather prominent in the lesson, as it was comprised of two stages and the class-time devoted to this activity was considerable. What we found from the analysis, though, is that its benefits were attributed as often to motivation and classroom-management (12 comments, e.g., "Pupils are only engaged in the lesson when they are given the task to create their own problem and their own story") as to epistemological issues (12 comments, e.g., "Thinking about own stories seemed to be very useful: why should I divide something in a ratio? [...] Creating a story, the pupils had to make various connections and grasp the meaning of what they are doing and calculating."). One reason for this could be the fact that pupil problem-posing is not an activity that is traditionally used in Czech classroom practice, and many students commented on the motivational strength of "novel" or "unusual" activity, because they simply viewed it as such. Ten out of the nineteen students who commented on this category offered an alternative to the activity management. Six students made neither a didactic nor a general pedagogical comment at all.

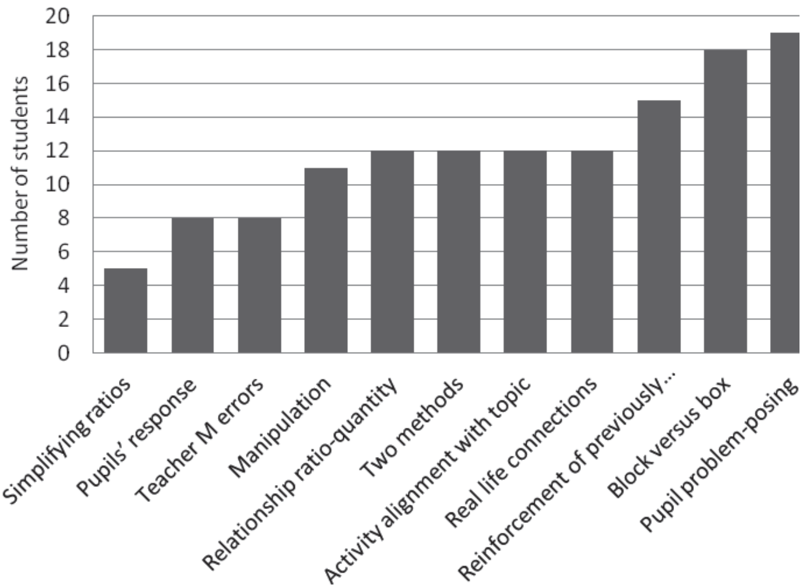

Figure 3 Individual categories noticed in analyses (adjusted for MS)

The other strongly represented category (60% of total students) was *Block versus box*. While blocks are counted as separate items, the empty boxes stand for a certain unknown number (or amount). Each must contain the same number (or amount). The letters *a, b* in the ratio *a* : *b* stand not only for a certain number of things but also for groups of (or boxes full of) things.

We used the following codes to measure the ability to notice the separate models in the sequence that the teacher uses to inductively introduce the lesson's topic:

Model 1: Each block represents a counting unit *and* a dividing unit/"share" (e.g., divide 12 boxes in the ratio 5 : 7).

Model 2: Each block represents a counting unit but *not* a dividing unit (e.g., divide 12 boxes in the ratio 1 : 2, "make them *equal piles*").

Model 3a: Each block represents a dividing unit (a share) and model 1 is applied ("How many boxes do you need if you divide 1 : 3?"). But now the idea of a box with some content is implied.

Model 3b: Each block represents a dividing unit (a pile, box, share) and is assigned (filled with) the same number of counting units.

Only a third of the students noted the transition between model 1 and 3b and in doing so they did not elaborate on the in-between stages. Seven of the students focused on the extension of the model 3b onto different types of counting units. Only one student distinguished the entire sequence.

The least noticed phenomenon was the one concerning the simplification of ratios. The opportunity to discuss simplifying ratios and/or the connection between ratio $a : b$ equalling $ax : bx$, where x is the amount of counting units in a share, comes up in the lesson on at least six occasions. Yet, the teacher seems to be avoiding the issue by performing the simplification herself whenever the need arises but neither addressing it directly nor mathematically explaining why she does (or does not) do so. Only 5 students noticed this concept at all, and only three of them commented on the teacher's not pursuing the topic. We can claim that this was the MS category that required the most advanced, "read between the lines" ability to notice.

Noticing pupils' responses and/or the teacher's work with pupils' responses also scored a low score (7 students reported on one or two particular situations, 1 student on six). In the expert analysis, we identified a minimum of seven major observable cases of teacher's handling a pupil response in a MS context.

The division of a quantity in a given ratio is introduced in the lesson using the model of cubes and boxes. This should help pupils to build an image of the whole process. The pupils first work with cubes and create ratios such as 1 : 2, 5 : 8, etc. Then they work with empty boxes. When solving problems, they are asked to first model the situation and only then to calculate. The aspect of manipulation was mentioned by 16 students (53%), however, only 11 (37%) mentioned the relevant process of modelling.

An interesting result arose in the case of interpretation of the final class activity (*Activity alignment with the topic*). Some students interpreted it as aligned with the topic of division of ratio, and some noted the switch to statistical data analysis. This case of contradictory interpretations can be accounted for by at least two factors: (a) the lesson was unusually long for Czech standards (students commented on this), so some students may have recognized the textbook-practice activity as a planned final stage of a unit and naturally viewed the remaining time as time for introducing a new topic, and (b) the lesson was long for critical viewing and it is likely that towards the end students' concentration dropped and they *assumed* the lesson's cohesiveness and the final activity to be working further with ratios, without taking the opportunity to examine the activity hand-out properly.

Finally, let us look into how students comment on what they see as negative (MS) points in the lesson, and whether they offer an alternative. Altogether 170 comments were coded as MS adjusted ones (76 were made by Group A students and 94 by Group B students). As a rule, in MS adjusted comments, a critical remark was accompanied by suggesting an alternative (or a correct mathematical answer in case of commenting on the teacher's mathematical errors). We found out that 28% (47 comments) of the total of MS comments were of a critical nature, including comments related to the teacher's mathematical errors (10 comments). Didactical alternatives for the teacher's action were included in 21% (37 comments). The alternatives regarded mostly a more efficient realization of activities (e.g., "The teacher gives her pupils time to derive the rule, in the end, though, doesn't let anyone explain, and conveys it to them herself."), including important mathematical aspects of the topic (i.e., the simplification of ratio), more elaborate work with pupils' responses, and alignment of the final stage of the lesson with the lesson's main topic.

Twelve students did not make any MS related critical remarks; on the other hand, four students gave an alternative in over one half of their comments. Figure 4 shows the representation of alternatives in all MS adjusted analyses. Naturally, the frequency of alternatives is, to a large extent, determined by the specifics of the lesson analysed.

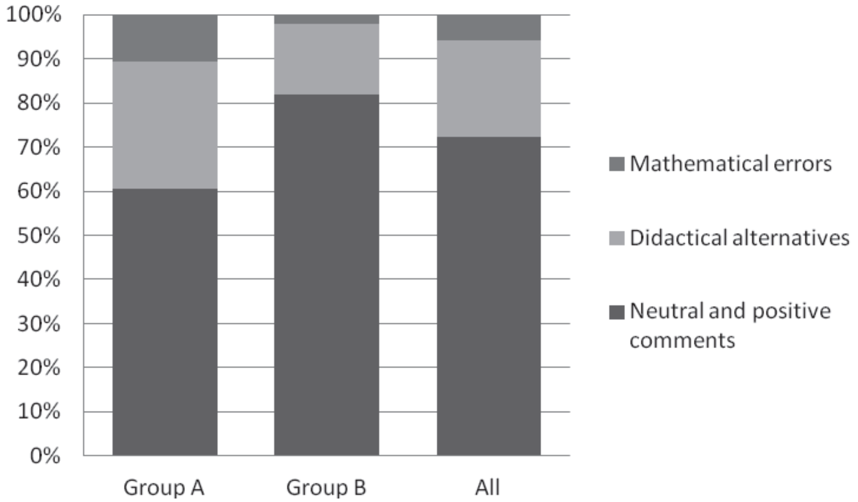

Figure 4 Neutral and positive comments vs. alternatives

When assessing the students' use of notions and terminology frequented in the theoretical groundings, the analysed text contained only five remarks with reference to ME theory, each made by a different student. These referred to the theory of generic models, which is a concept development theory (Hejný, 2003).

Our investigation of differences between students in the first course of ME (Group A) and students of the second or third semester courses (Group B) did not yield any conclusive results. In terms of the ability to notice, the different MS phenomena, two categories (*Teacher's mathematical errors* and *Simplifying ratio*) stood out, however, we do not believe that the amount of data involved is enough to assure statistical significance. Perhaps one distinction between these two groups was apparent and seems to be of interest here: we noticed that the propensity to provide didactical alternatives and correcting teacher's mathematical errors was stronger in Group A (where 40% of all MS comments were such alternatives, compared to only

18% of total comments in Group B). Figure 4 shows the difference between the two groups. This difference may be an occurrence particular to our group of participants. It can also be conjectured that students with hands-on teaching experience (i.e,. student-teaching) are more empathetic with the observed teacher. Nevertheless, such hypothetical causality would need to be explored further on a larger set of data.

5 Discussion

It is important to note here that the studied analyses were rich in comments concerning non-MS (i.e., especially generic and classroom management) issues. Our study was conducted from a perspective of the belief that perceiving classroom situations mathematically is an important aspect of teaching practice. Focusing on MS phenomena enabled us to look more deeply into the nature of what in MS phenomena is being noticed or omitted.

It has been widely acknowledged that a teacher's mathematics knowledge is only one of the pre-requisites for teaching practice conductive to mathematics learning. Measuring how much our students have been able to observe informs us on how aware they are of the complexity of the mathematical skills required of a successful teacher (in other words, how developed their PCK is). This study confirmed the results of research presented above in that student teachers often neglect the mathematical aspect of teaching situations. In particular, the following elements of instruction can be identified as strongly dependent on PCK:

Introducing a topic

In the lesson at hand, the first (inductive) stage required a careful examination of the target topic (i.e., a chosen set of concepts)[3] – dividing a quantity in a given ratio – in terms of its inner structure and characteristics, considering the "building blocks", e.g., the concepts of ratio, divisibility (a consideration included in the category *Relationship between ratio and quantity*), equal parts and irreducibility (as in the category *Simplifying ratios*). The individual models are exemplified and enacted using manipulatives. Yet, from the students' analyses it perspires that this carefully devised series of graded tasks and examples in the inductively led introduction of the topic itself went unnoticed by the majority of students. Although it can be assumed that they themselves have the knowledge to perform the division of a quantity in a given ratio (that is, the content knowledge), they do not appreciate the process of breaking the topic down and examining the fine points (their PCK is insufficient in this area). They only notice and comment on what they see on the surface, i.e., the fact that the teacher uses blocks and boxes. There is no significant difference between the two groups of students. The reason might be that the students do not

3 It is clear from the teacher's lesson plan, which is available for the lesson in question, that the introduction of new material is the bulk of her preparation and that she examined the sequence of models in depth.

have enough experience with preparing an introduction of subject matter based on the inductive method (which has a pedagogical implication for their ME courses).

Choosing relevant instructional activities and providing various representations

The other level of mathematics specific awareness is the target set's place in the structure of mathematical concepts (most importantly, but not exclusively, in relation to curriculum, i.e., considering questions like these: What do my pupils know already, how does relate this new topic to that knowledge? What do I need to review/activate, focus on? Are there other ways of solving the problem? What other concepts might come up during the lesson and in what context? Which ones do I want to explore? How do my activities relate to future learning?) as well as its applications in other subjects or areas of real-life experience (Why is it important for my pupils to be able to do this part of mathematics? How does this mathematics relate to their personal experiences? Which applications offer a new way to represent the concepts?). In our specific lesson, this knowledge comes through most prominently in categories *Pupil's problem posing*, *Two methods*, *Teacher's reinforcement of previous knowledge*, and *Activity alignment*. Again, if we look at the students' written analyses as indicators of this MS knowledge applied to this lesson, we see that although these categories represent the more noticed ones, they are ignored by the majority of students (with the exception of *Pupil's problem posing*, as discussed earlier).

Working with pupils' answers

The ability to understand, interpret and react to pupils' responses and notions is highly dependent on PCK. This is, of course, on two levels, one of them is to notice the answer and one is to notice the teacher's handling it, but they are actually in one, as a non-descriptive remark on the first leads automatically to handling it in most of the cases we observed in our analysis. Our students' low performance on this particular subject of attention aligns with the results of others (e.g. Star & Strickland, 2008). It has been suggested by Gall and Acheson (2010) that noticing the pupils and tuning into their learning processes is one of the further stages on a teacher's learning path.[4] Spangler (2011) stresses the importance of training student teachers in reading and reacting adequately to pupils' responses and errors. Our own study results further confirm these points.

Next, let us summarize some other results from the presented study. In their above research, Santagata, Zannoni, and Stigler (2007) found out that the participants' comments in the pre-test were mostly positive and only after the course, they "assumed a more critical approach when watching the lesson. […] In the post-test, participants re-evaluated some of their observations. They noticed some contradictions in the teacher's actions. They reflected on what they observed, discussed

[4] Of course, the nature of the video recording may have also contributed to the lack of students' attention to pupils' learning – the video-camera mostly focused on the teacher and the class as a whole.

possible problems, and often proposed alternative actions." In our research, we got contradictory results. As shown above, our students were quite critical to the teacher's action and proposed alternatives, and more strikingly, the less experienced group of students did so more frequently than Group B. This can be perhaps explained by a certain level of compassion towards a teacher and a reluctance to criticize in Group B (as the students can better relate to the teacher as their future role), or by being able to stress the positive aspects of viewed material.

The authors who use videos in their courses (some references given above) claim, among others, that the video is a way to connect the theoretical knowledge taught in the courses and practice. However, we were disappointed to see that our students in Group B did not use their theoretical knowledge taught in the ME courses for the description and interpretation of the lesson observed. This has an important pedagogical implication for the ME courses – tasks must be developed which explicitly ask for the description of some pedagogical situations in terms of the theoretical concepts. It seems that the connection of the theory and practice is far from straightforward.

Finally, as stated above, both groups of students had had an experience with analysing observations of lessons from the point of view of generic aspects rather than mathematical. Group B students also took part in a teaching practice in mathematics during which they were provided (mandatory) opportunities to observe experienced mathematics teachers and to reflect on these observations. But, as we can see from our study and the study of others, too, this learning is far from self-evident. Star and Strickland (2008) point out that student teachers often observe lessons as learners of mathematics, not as mathematics teachers: "The kind of observing that one does as a learner typically concerns the comprehension of the presented material (e.g., Do I understand what was just said? Does the mathematics make sense to me?) and does not prompt the observer to think deeply about the teaching and learning process more generally." Thus, we believe that even the 'ordinary' ME courses not only the ones which are organised around videos should include activities which aim to develop student teachers' ability to notice MS phenomena.

To sum up, in this study we devised a method for measuring mathematics education students' ability to notice and comment on *mathematics specific phenomena*. This method is based on analyzing students' written reports on a specific lesson. Although we are aware of the limitations of the scope of our study for making significant conclusions (the data is not extensive enough and the factors involved are too many), in terms of student experience with ME environment and/or exposure to ME course material, we hope these results will inspire future investigations. Perceiving situations mathematically can be a matter of training (van Es & Sherin, 2002). It is important when the students are in the role of a teacher: they should be able to react in such situations without losing track of the mathematics behind it. For example, being able to introduce a topic effectively, choose relevant activities and analyse pupils' misconceptions or work with pupils' approaches to problem-solving.

The research presented in this paper forms part of a wider study aimed at the student teachers' ability to notice MS phenomena when observing a video recording of a lesson or its segments. Future analysis of data will be conducted in the effort to shed light on individual students' development of ability to notice these phenomena across time and ME course attendance, on the effect watching selected segments (rather than a whole lesson) may have on commenting on MS, or on identifying general trends in the nature of MS aspects that are noticed or missed by pre-service mathematics teachers. A comparative study on in-service teachers could also bring relevant findings and deepen the understanding of this issue as well as provide important evidence to guide the design of pre-service teacher development programs, and potentially help link the ability to notice specific phenomena to effective teaching practice.

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