

# Should Learning (Mathematics) at School Aim at Knowledge or at Competences?<sup>1</sup>

Miroslav Rendl, Stanislav Štech

Charles University in Prague, Faculty of Education

**Abstract:** The Czech school reform tends to aim at competences rather than at knowledge. We attempt to bring some evidence that the *mathematical competences* is a vague term without precise scientific as well as practically useful contents. Based on the analysis of the Czech data from PISA 2003, we could not substantiate the differences in teaching practices referred by students with different mathematics test results, i.e. allegedly with different levels of math competences. Thus, any more or less effective competence-teaching practices could not be identified. Similarly, in TIMSS 2007, we did not find any differences in teaching practices between the Czech “pro-reform” and “antireform” teachers.

We further investigated the reformists’ requirement to set up the real life, on the one hand, a starting point of the school teaching/learning, and, on the other hand, its ultimate goal. In the studies exploring the issue, we found no definite results in favour of the use of real-world problems in school mathematics. Similarly, the effectiveness of any particular method or practice of school knowledge acquisition on a far transfer into the everyday or even professional life cannot be proved. Moreover, the usefulness of the concept of transfer itself was questioned in recent debates as redundant (existing besides the concept of learning).

We conclude that the Czech reformist discourse is predominantly based on extracting partial moments of the teaching/learning process, putting them as antagonisms and labelling them as universally good or bad. On the contrary, we argue that these different moments, however opposite they could seem, stand as mutually linked parts in the real teaching/learning process.

**Keywords:** knowledge, competence, teaching/learning process, real-world problems, transfer, school reform, reformist discourse

For more than twenty years, the educational discourse in the Czech Republic has been engaging in debates – sometimes implicit – focusing on an issue which can be summarized as follows: is the child’s learning more efficient if he/she appropriates items of knowledge (mostly identified with memorizing and drill) or if he/she constructs, discovers, leads inquiries or solves problems? The first orientation tends to be termed “passive”, whereas the second “active”. This controversy appears in educational practice and theory under the label of *traditional* teaching/learning or teaching/learning based on *direct instruction* vs. *constructivist*, *competence* or *skills* oriented teaching/learning.

---

<sup>1</sup> The study was carried out within financial support from the Grant Agency of the Czech Republic, by Grant No. P407/11/1740 “Critical areas of primary school mathematics – analysis of teachers’ didactic practices”.

The reformist discourse in the Czech Republic, as in many other countries,<sup>2</sup> advocates the latter position. In striving to answer the question of effective teaching/learning, it has set up a stark opposition between these different approaches to teaching. The experts of the National Institute of Education, for instance, express the crux of the curricular reform (and, more generally, the reform of teaching/learning) in the following, succinct way: “Simply put, students should no longer be required to learn heaps of encyclopaedic information. Instead, they should learn to think logically, search for information and use it. They should be encouraged to be more active in class and to deal with problems independently. ... Graduates should not only become experts in the given area but should also acquire the so-called key competences – the ability to communicate, co-operate, bear responsibility, make themselves understood in a foreign language, use information technology. Furthermore, they should be willing to pursue life-long learning. This will be essential in their future lives, as they will be required to respond to social and technological changes.” (Franklová, 2009, p. 22)

This somewhat Manichean treatment of the problem results in a series of antinomies which, however, are not evidence-based. In the present article, we will describe and discuss in greater detail the (school) knowledge vs. competence (for life) controversy and the extent to which such an approach can be substantiated by the research evidence in the field of education.

Actually, this debate revolves around the following issues:

(1) With an appeal to the decline of the results shown by Czech students in PISA and TIMSS surveys, reformists allege that school teaching/learning (including the teaching/learning of mathematics) is inefficient. School teaching/learning should focus on the *development of cognitive competences (of thinking skills)* instead of *subject matter knowledge*, they say.

(2) The PISA surveys assess *competences for life*. Therefore, it is said, we must set *real world-problems* – instead of de-contextualized “abstract” knowledge – as the starting point of the teaching/learning.

(3) They believe school teaching/learning focused on competences to be more efficient since it allows for a long-term (far) *transfer* – which is meant to be in line with the alleged general long term aim of school teaching/learning: the practical application of its outcomes in life.

The question that needs to be raised is the following: which approach or method of teaching/learning is preferable? Is a general answer to such a question available at all – irrespective of the cognitive level of students and the potential of their proximal de-

<sup>2</sup> The benefit knowledge acquisition may have for the intellectual development of the student is periodically questioned in many countries. The most marked example is the USA, where blanket evaluations of students’ school results show that despite the improvement in basic competences, the students still do badly in higher learning processes. This leads to repeated criticism levelled at the school for its not being able to ensure that students will acquire the methods and competences related to thinking skills. The teachers are subsequently pressed to focus their teaching on thinking competences (de Bono, 1976; Ennis, 1987; Paul, 1993). The implicit opinion is commonly shared that the school puts too much emphasis on knowledge acquisition, while it should preferably pursue the development of intellectual competences applicable in practice.

velopment, or regardless of domain of learning or of the nature of the task? And what kind of educational research evidence is supposed to help us in such a deliberation?

## 1 Are Competences the Goal to Which the Curriculum is Just a Tool?

From the reformists' point of view of the education, the curriculum is a subject serving other, more primary objectives. Within the Czech curricular reform, its role is "subservient" or instrumental because students' competences are the goals. This is apparent from the wording of the policy reform document called *Framework Education Programme for Basic Education* (RVP ZV, 2005) and it is even more obvious in its further interpretation. For instance, J. Maňák states that the reform consists in a radical change of objectives and of the content of education: "Elementary education should help the students to gradually develop key competences and provide a reliable basic general education, oriented especially on the situations close to life and practical acting. These are important target values not previously delimited in such a pregnant and univocal way. ... The curriculum is understood as a tool that has to lead to the achievement of key competences" (Maňák, Janík, & Švec, 2008, p. 35). The subordinate role of the curriculum is further revealed in the structure of the text of the RVP ZV, which puts forward nine very broad educational areas followed by educational fields. Only in the last step attention is paid to school subjects and their curricula. The document relates these levels within a hierarchic order: particular knowledge is included by way of curricular *example* only after target competences in the educational area, field and subject have been delimited.

The quite indefinite term competence (construed as a "system of knowledge, skills, abilities, attitudes and values that are important to the individual's personal development and to the individual's role in society", RVP ZV, 2005, p. 14) comes, as Ropé and Tanguy (1994) show, from the sphere of vocational (professional) education. The conception and instruments of vocational education are meant to provide the model for all education, from pre-school to university level. Laval (2004) explains that the term "competence" is neutral only in appearance: it seemingly expresses a universal psychological construct relating only to an individual and his/her acquired, largely implicit qualifications, not controllable by the school. This, however, obscures the fact that it involves a very unclear connection between theoretical knowledge and skills on the one hand and of practice on the other. Laval thus reminds us that with the introduction of *competences*, we are led into the world of evaluation tools, control, supervision and of a gradual search for maximum rationality. But this world is the world of out-of school, especially of work, in which a set of required processes and skills needed to fulfil particular operations is established and adapted ad hoc to meet the criteria of profitability. The competence is ultimately destined to serve a practical, utilitarian purpose. Hence, at the heart of the term competence, one finds the criterion of being "close to life and to practice".

Nevertheless, the criterion of utility for practical life is gaining ground even with-in the critique of the school form of teaching/learning. For instance, the reformists' critique extends even to the teaching of mathematics which, in its "traditional" form, allegedly ignores the natural, everyday life of the child. They believe this serves to undermine students' motivation: "After all, as early as the first class, the renaming of the subject itself from "Sums" to "Mathematics" and the compulsory use of the terms "plus" and "minus" instead of the until then common terms "and" and "without" bears witness to the tendency to ignore the natural connection of the child to his/her concrete world" (Mleziva, 2011, p. 5).

Yet PISA itself states that the focus of mathematical tasks was on the "functional use of knowledge in solving real-life problems, rather than on ascertaining to what degree they had mastered their studies of formal mathematics or the degree to which they were facile with particular facts or procedures" (*Learning Mathematics for Life*, 2009, p. 36). And in defining mathematical literacy, the ability of students to solve real-world problems with the use of skills and knowledge gained both through school teaching and in everyday life is posited as a criterion of successful teaching. Competences should thus serve both as the starting point and as the objective of teaching: "Mathematical literacy is about dealing with 'real' problems. That means that these problems are typically placed in some kind of a 'situation'. In short, the students have to 'solve' a real world problem requiring them to use the skills and competences they have acquired through schooling and life experiences." (*Learning Mathematics for Life*, 2009, p. 20)

The data of both PISA, and partially also of TIMSS, may therefore be expected to give us a more precise idea of the contents of the term "competence" and of the kinds of teaching that influence – positively or negatively – the development of such competences.

Czech reformists often refer to the PISA and TIMSS surveys in two different ways. On the one hand, they, in accordance with PISA, emphasize that the objective of the reform consists in the development of "competences for life instead of knowledge". On the other, they explain the bad results of the last surveys in the following way: Czech students lag behind because the reform has not been implemented consistently and has not been carried out fully yet.

Our question is whether it is possible to identify in these surveys any differences in Czech teachers' ways of teaching and whether some of these ways lead to better student results than others.

During the TIMSS 2007 survey, Czech teachers were, among other things, asked to answer the following question: "To what extent do you agree with the following judgements concerning the reform in progress: a) I support the reform; b) the implementation of the school educational programmes<sup>3</sup> (SEP) is just an outward change?" This allowed us to construe a group of "pro-reform" teachers that agree ("definitely" or "rather") with the first statement and disagree with the second,

<sup>3</sup> The school educational programmes should be a concrete tool by means of which the curricular reform privileging competences is realized in the given school.

and a group of “anti-reform” teachers who answered in the opposite way. On this understanding, 15.1% of the teachers in question were in favour of the reform, 61.5% of them were against.

When comparing the groups of teachers, we focused on whether there were any differences in their students’ overall results in the mathematics test. The difference in the national Rasch score (mean 150, STD 10) was 2.47 in favour of the pro-reform teachers. But even this small difference could be attributed to the fact that a disproportionate percentage of the pro-reform teachers from the Czech sample teach at eight-year grammar schools where the performances in the test are generally higher in comparison with the corresponding grades at basic schools. When a separate comparison was made between pro-reform and anti-reform teachers first in secondary basic schools and then in eight-year grammar schools, the differences in results were lower than 0.5 point.

Furthermore, we compared these teachers based on the methods of teaching they use in their classes.<sup>4</sup> When we compared their answers, some differences became apparent. The pro-reform teachers purportedly frequently ask their students “to apply facts, concepts and procedures to solve routine problems”, “to explain their answers”, “to work together in small groups” and, on the other hand, do not often require them “to relate what they are learning in mathematics to their daily lives”. Even though some of the differences could seem surprising, they were immediately challenged when the same comparison was made based on the answers of students. From this point of view the pro-reform teachers differ from the anti-reform mostly in that in class, students “use calculators”, “begin their homework in class” and “explain their answers” more often, and that, on the other hand, they “practice adding, subtracting, multiplying, and dividing without using a calculator”, “review their homework” and “work on fractions and decimals” less often. We can see that there is very little overlap between the answers of teachers and students – they agree only in that students are required to explain their answers more in classes taught by pro-reform teachers.

We also wanted to find out, whether particular answers given to question 17 (teacher questionnaire – TQ) and to question 10 (student questionnaire – SQ) influence the overall performance in the mathematic test in any given way. At first, we calculated the correlations (Spearman’s rho) with the overall result for individual items. The correlations were very low or negligible.

We also examined whether a certain combination of the items had a stronger influence on test results than individual items. We carried out the factor analysis individually for the items in question 17 (TQ) and in question 10 (SQ). For both of the questions, a two factor model was suitable for the description of the connections between the items. Moreover, for both of the questions, only one of the factors was saturated by the differences between “pro-reform vs. anti-reform teachers”. The

4 Teacher questionnaire – question 17: “In teaching mathematics to the students in the TIMSS class, how often do you usually ask them to do the following?” (items *a-l*)  
Student questionnaire – question 10: “How often do you do these things in your mathematics lessons?” (items *a-q*)

28 contents of most of the other items saturating any of the factors allowed for the hypothesis that these are the factors characterising, to some extent, the reform and, on the other hand, traditional teaching. So we used this result to create the aggregation of items in both questions 17 and 10. But even the correlations between the aggregated items and the overall mathematics test results were negligible (the highest was  $r = 0.085$ ).

On the other hand, we found an interesting correlation of the test results with the school grade in mathematics ( $r = 0.605$ ). Evaluation using grades is often criticised by the reformists as inadequate and as a formal expression of success at school that leads to undesirable types of motivation for the children and to the acquisition of knowledge which is merely formal and precludes deeper understanding.

In the PISA 2003, we were not given the possibility of comparing two predefined groups of teachers because no questionnaire for teachers was administered in this survey. Instead of this, we started with the assumption that the test result itself is an expression of competences and that the ways of teaching that would significantly correlate with the test results could be considered as pertinent in shaping such competences.

The question Q38 of the students' questionnaire does not give a detailed account of ways of teaching in class, unlike the TIMSS 2007. Nevertheless, for some of the items it could be assumed that they may be connected with the reformist mode of teaching and thus influence the results of the test.<sup>5</sup>

Similarly to the TIMSS, we found negligible correlations between the answers to the question Q38 and the overall test results and a much higher correlation of the item 39a, which asked about the mathematics school grade in the most recent school report ( $r = 0.425$ ).

The correlation of the grade and the PISA test result is thus lower than for the TIMSS test. But it is questionable whether it confirms that there should be an important difference between the nature of the TIMSS and PISA tests, or, whether the difference corresponds to the difference between mathematics knowledge and *mathematics* competences.<sup>6</sup>

Whether we consider the result of the test to be an indicator of the level of knowledge or of the level of competences, we come to the conclusion that, bearing on the data of PISA and TIMSS, there is nothing to deduce about the efficiency of ways of teaching. It also is not clear from the data which ways of teaching allow us to distinguish between pro-reform and anti-reform teachers, or between teaching of competences and the traditional teaching of knowledge.

<sup>5</sup> Student questionnaire – Q38: How often do these things happen in your <Mathematics> lessons?

<sup>6</sup> The question arises here to what extent the statement that PISA assess the ability of students “to use their mathematical knowledge in solving mathematical situations presented in a variety of settings” (Learning Mathematics for Life, 2009, p. 36) is valid and to what extent focus shifts from solving mathematical situations to the quality of the description of the context, the frequency at which the student comes in contact with that context and its proximity or distance from the school context. The authors underline that the ability to switch between the school context of using mathematics and its use in various everyday contexts is taught – to a different extent in various countries.

## 2 “Real-World Problems” (RWP) as a Starting Point of Learning

If the competences are delimited at all, their key feature should consist in their relation to real life, in the two following senses:

1. contexts of everyday life should form the starting point of learning,
2. the general long term objective of school learning should be the use of its results in practice, in real life – this presupposes their transfer far in time and context.

In this section, we will discuss the first issue. The question of transfer will be discussed in section 3.

The popularity of teaching based on everyday life problems was augmented by the development of the so called situated learning (Lave & Wenger, 1991). The situated learning approach had been initiated by the justified criticism of the application of intelligence tests coming from the western civilization area to people from other cultures. This is because successful solving of tasks in these tests requires ways of thinking and knowledge gained by systematic school learning. But when these were applied to populations with significantly different cultural background or to culturally disadvantaged (ethnic minorities in the USA or indigenous populations in Africa), their results were very poor. This led researchers to the idea of monitoring the skills of these people in their everyday life. The positive result was the discovery that the tested people are not intellectually retarded, because in situations that are familiar to them they master the required cognitive operations and skills (Cole, Scribner, 1981).

Significantly, the situated learning translates into French as ‘learning in context’ (*apprentissage en contexte*), a somehow inaccurate expression, but one which makes explicit reference to an important dimension of situated learning, that of context, which in turn reminds us of the other necessary term of the relation, “text”, making salient the “text–context” relationship. This turn towards situated learning, towards forms of cognition and learning in practical situations (e.g., intellectual operations developed by tailors apprentices in Jean Lave’s experiments in Liberia, 1977), towards learning in practice (e.g., everyday arithmetic in the research conducted by Scribner, 1986, and by Rogoff, 1990) led to a full appreciation of cognition as a set of cultural practices and of the impact of the context on learning. And the term “apprenticeship” was established to denote the learning embedded in a (real-life) situation (contrary to the term “learning” denoting the formal school learning).

At the same time, it may have led to the overestimation of this form of learning at the expense of the importance and function of school forms of cognition and learning, of what we could call the “text in the situation”. In conjunction with the reviving educational reformism and a return to student-centeredness, this led to the overall negation of the developmental significance of school forms of cognition. Situated learning in contexts of practical life of the individual was placed on a pedestal, almost as a model for learning at school. Reformist (student-centred) conceptions of teaching/learning are strongly nurtured by this conception (Štech, 2008).

Many users of situated cognition theory are insensitive to the fact that learning at school is also learning in a context with its own specificities, a context which represents a community of practices largely derived from a concept of scientific knowledge. A comparison with extra-curricular contexts makes it evident that the objective of school is epistemic. It aims at the transformation of modes of thinking, of experiencing, and of the self. This requires a clear conception of the relations between spontaneous learning (the kind of learning we do, and what it is we learn, in everyday contexts) and education, formal learning and development. School teaching contributes to the so-called intellectualization of cognitive operations – to working with principles with economical abstraction, which brings a different quality of attention, memory or thinking and “raises” the spontaneous thinking in everyday practical situation to a higher level. Let us show the difference on one example.

Carraher and Schliemann (2002) refer to some examples of thinking of adults who did not attend school and to limitations of their knowledge gained in everyday practice:

“We asked Brazilian school children and street sellers who had received little or no instruction on multiplication to solve aloud pairs of verbal problems where they had to compute the price of a certain amount of chocolates based on unit prices (Schliemann, Araujo, Cassunde, Macedo, & Nice, 1998). The following is an example of the problems pairs we used: Type 1: A boy wants to buy chocolates. Each chocolate costs 50 cruzeiros. He wants to buy 3 chocolates. How much money does he need? Type 2: Another boy wants to buy a type of chocolate that costs 3 cruzeiros each. He wants to buy 50 chocolates. How much money does he need?”

Participants first solved a problem where the larger number denoted the price of one item and the smaller one indicated the number of items to be bought. Immediately after they were given the corresponding problem where the smaller number denoted price and the larger one denoted number of items and were asked whether they knew its answer without doing any computation. If they used the former problem to answer the latter, we took this as an indication that they relied on the commutative property of multiplication. The group of school children who had received school instruction in multiplication (second- and third-graders) solved the first problems in each pair via multiplication and frequently relied on the commutative property to answer the second problems. In contrast, street sellers tended to use repeated additions throughout and rarely invoked the commutative property to answer the second problem. Instead, they successively added the number of cruzeiros, a cumbersome procedure leading to frequent errors if they had to add, for instance, 3 cruzeiros 50 times.

The above results suggest that, although people can learn meaningful mathematical ideas in mundane, non-academic situations, they nonetheless need access to new symbolic systems and representations they are not likely to acquire out of school.” (2002, p. 253–254)

The issue of effectiveness of the use of “real-world problems” (RWP) in school teaching/learning is also dealt with in the National Mathematics Advisory Panel



(NMAP) report. It states that, especially, this term is used in a very diverse way: “A serious problem in synthesizing the research in this area is that there is no clear, agreed-upon meaning for “real-world” problems” (NMAP, Report of the Task Group on Instructional Practices, 2008, p. 98). In comparable studies on contextualization, the use of “real-world contexts” in school problems proved effective, but only if these were contextualized in a similar way. Regrettably, the students did not manifest the improvement in other, more general skills (computation, simple word problems, and equation) beyond the solving particularly contextualized problems. Other studies, which unfortunately showed some methodological flaws, mostly found positive effect of learning on contextualized problems, but the effect was measured on RWP the control groups have not learned (NMAP, *ibid*, pp. 96–98). The effect of teaching based on RWP, therefore, does not reveal to be the consequence of the particularity of the problem originated in everyday life, but rather of the fact that the context taught in experimental groups was new, not commonly practised in the school and – paradoxically – unknown to the students of control groups in connection to mathematics.

These results are very similar to the results of studies on transfer we will mention in the following section. In most of the studies on various teaching practices that should increase the transfer, the impact is predominantly observed on the *near* transfer. The effect is found in contextually close domains and in relatively short term. Long-term transfer to problems with more different contexts was not proved.

The studies discussing the effect of RWP emphasize more the dragging the students into the context or into the story of the problem as a parameter of efficiency rather than using the elements of the non-school environment just as props. Regarding this, we doubt the PISA problems – given to the teachers in the Czech Republic as an example of “teaching for life” – meet this condition. At least, it is evident that it is not possible to set a problem that meets such a requirement universally for all the students of the given country.

The ambiguity of the conclusions of the studies about the effectivity of RWP rather corroborates the opinions of Son and Goldstone (2002). They report on three experiments on contextualization they conducted. In these experiments, the control groups always had better results in the tests than the experimental groups learning under the conditions of various types of contextualization. The authors then come to cautious conclusions:

“Our experiments suggest that it is not that concrete experiences, activities, and demonstrations are generally good or bad for transfer, but rather these manipulations cause particular construals that affect learning and transfer” (*ibid*, p. 75).

Our results should not be taken as opposing contextualization, personalization, or learner adapted approaches. Instead, our experiments show that the ‘one size fits all’ approach, where personalized contexts are simply grafted onto contents, could have negative cognitive consequences. Undoubtedly, participating in activities and evoking real world knowledge is influential and can result in effective activity-specific encoding. For example, warehouse drivers in a dairy organize information

**32** according to warehouse location and pallet size whereas consumers typically encode by general categories (Scribner, 1985). Their activities and perspectives allow them to selectively encode relevant information. However, this context-bound encoding leads to potential pitfalls. (...) A de-contextualized understanding may be beneficial for learning structural principles by leading to a broad, detached understanding of a situation rather than being guided by a particular perspective” (ibid, p. 76).

It appears, therefore, that to base education on problem tasks may have a certain importance at a given stage of teaching/learning or for a given sub-class of students. Nevertheless, their positive effect is by no means to be expected to apply universally and it makes no sense to understand the issue as that of two mutually exclusive types of approach.

May it be, though, that real world problems and the ability to handle them serve to prove whether the ultimate objective of teaching/learning has been achieved? Let us have a look at what research has to say about real world tasks as a criterion of the effectiveness of teaching/learning.

### **3 “Problems in the Real World” as the Objective of Learning**

From the point of view of psychology, the ambitions that the school learning should be the “learning for life” corresponds to the issue of the far transfer – an issue studied for more than one hundred years. The classical concept of transfer has been based on the notion of identical elements: the transfer happens thanks to the correspondance or overlapping of elements contained in the learning contents and in the target situation. Cognitive revolution reworded this in the sense that the transfer happens when the mental representations of the original and the target situations are identical or overlapping (Lobato, 2006, p. 433).

This classic interpretation was criticised during the whole 20th century, so that some authors state even after a hundred of years the results of research are still ambiguous and the problem of transfer remains open (Barnett & Ceci, 2002; Wagner 2006; Lobato 2006).

Barnett and Ceci (2002) see the basic problem of unsatisfactory results in divergent interpretations of transfer by researchers. This makes comparability of their results rather impossible. Therefore, even though there are many studies on transfer, they infer it is not possible to come to more general conclusions.

That is why they suggest the taxonomy of transfer based on two global dimensions – that of content and that of context. The dimension of content is further divided according to the following criteria allowing to classify the transfer: 1. the skill to be learned, 2. the nature of the expected change, and 3. the requirements on the memory (whether there are prompts available and whether the nature of the process is more a recall or a retrieval). The dimension of context consists of six sub-dimensions: (1) the domain of knowledge, (2) physical context (e.g., in school

vs. out of school), (3) time context (near vs. far transfer), (4) functional context (different from the physical one, but the distinction is not quite clear), (5) social context (learning alone or in cooperation), and (6) modality (oral or written, visual or auditive transfer, etc.).

They think about each of the sub-dimensions in two values (near – far transfer) defined as the distance between the learning situation and the situation which the learned skill should be transferred to. But even with this limitation, the articulation of context according to these dimensions leads to 64 combinations. Even after reducing the number of dimensions to the two or three most important ones, the authors found only very few studies (often just one) for every cell of combination matrix, so that it was not possible to make any comparison. But they were not even able to classify many of the studies according to their taxonomy, because these did not provide the necessary description of the methodology.

Barnett and Ceci (2002) consider their taxonomy to be, on the one hand, a tool that could measure the comparability of studies and on the other hand, to be an appeal to fill the empty cells, i.e., the missing combinations of context dimensions by further researches. Just as an example, they have not found any study testing the transfer to the far knowledge domains, far physical context and far time. This finding is a very interesting one as this is just the kind of transfer the reformists expect the school teaching/learning should guarantee.

Nevertheless, most of the authors criticizing the traditional research on transfer disagree with the way the transfer is conceptualized. It is a seemingly very broad criticism using many partial re-conceptualizations and also many new terms. But we still think that it is possible to summarize it into three basic issues.

1. Most of critics point out that it is necessary to *understand the transfer dynamically*, not statically. The acquired knowledge is not transferred to the new situation directly as a priori prepared, fixed and abstract schemes. In a new situation, the learner explores at first what from the previously acquired knowledge could be used to solve it. Thus, the knowledge changes in the process of transfer, adapts to the new situations and enter into new combinations: "... there is little evidence for some monolithic skill or piece of knowledge being carried over intact from a unique prior situation to the present one. On the contrary, the students are wrestling with multiple, competing ideas. (...) They do far more than deploy this knowledge. They draw upon it selectively to deal with the unique predicaments at hand. They have not simply unloaded a prior solution from their storehouse of knowledge. They have crafted it on the spot, adjusting and adapting their prior knowledge in the process" (Carragher, Schliemann, 2002, p. 19)<sup>7</sup>. From this point of view, it is impossible to determine in advance what will be considered as a transfer or as its criteria. But this is just the way the quantitative researches are conceived according to their randomized controlled trial (RCT) ideal as they attempt to measure transfer in terms of the improvement of the performance defined in advance.

<sup>7</sup> Similar description of the learning process could be found in the article by Wagner (2006).

On the other hand, qualitative studies do not start with an idea of the result of the predefined transfer, but make the content of the transfer their object of research. "... evidence for transfer ... is found by scrutinizing a given activity for any indication of influence from previous activities and by examining how people appear to construe situations as similar using ethnographic methods, rather than relying upon statistical measures based on improved performance" (Lobato, 2006, p. 436). Also the final part of the Wagner's article can be, in fact, understood as an appeal to use such research procedures that are inherent to the ethnographic approach. Especially, he stresses the necessity to include the manner the student himself/herself sees and processes the problem into the researcher's analysis of the process of "transfer in pieces" (Wagner, 2006, p. 68).

Even though Lobato states that it is necessary to distinguish between the criticism of methodology in the transfer research and the criticism of its theoretical concept, it seems that this can hardly be achieved. The "new" concept of transfer clearly requires data that could be obtained only by the qualitative methodology. Lobato does not pay attention to this necessary requirement; in fact, she complies with it only verbally. Her actor-oriented approach – sometimes she even uses the unclear expression of actor-oriented transfer – is nothing else than the ethnographic approach to the research of transfer.

The qualitative methodology, on the other hand, clearly does not provide the data inherent to the experimental and quasi-experimental research. Does it make sense to prefer one of these to the detriment of the other?

2. The criticism is also aimed at the classic idea that the far and flexible *transfer* is ensured by *mastering abstract schemes* and concepts. In this vein, Wagner objects to such an assumption that, at least at the beginning of a learning process, no fixed mental representation is created and that there is nothing that could overlap and with which the new situation could be compared. We can see a certain logic in his opinion: to search for overlapping with just one concrete (previously created) mental representation and not with another one, the student would have to categorize the new situation (problem) under the more general category of similar situations. But the learner cannot be aware of such a category at the moment.

Nevertheless, this is true for the unguided problem-based learning only. However, in Wagner's discussions with Maria, one of the participants in his research on probability problems (Wagner, 2006), we can see something slightly different. In solving the problems asking for the most appropriate number of coin tosses under various conditions for winning, there was a great positive change in answers and comments after Maria learned about the law of large numbers and central limit theorem in school.

Apart from other things, this clearly shows how the classroom context of school subjects curriculum (the matter "we just go through") originates the awareness of the fact that the next problem to be solved probably belongs to the same class of problems. This generates the basic frame of inter-contextuality – still in external form that has to be internalized and further developed. Here, Wagner, but also oth-

ers, set up a too sharp (antagonistic) contrast between the mental representation of a class of problems (schemes) and the mental representation of a single problem or situation which Wagner calls a collection of pieces of knowledge and particular schemes. The way Wagner describes the deepening of the understanding and what he considers to be the process that leads to the abstraction<sup>8</sup>, could be described as building and deepening of the mental representation as well, the germ of which comes of the internalization of a scheme or algorithm provided by the teacher's direct instruction. Here, in fact, we face the old and well known problem: the tendency to excessively stress the difference between the bottom-up and top-down process.

Finally, the transfer is a matter of the process of understanding abstract knowledge (terms, algorithms, schemes). Understanding is never a one-shot event; it is a long-term process where the bottom-up and top-down approaches alternate and the understanding becomes gradually deepened.

3. The traditional research on transfer gets criticised for conceiving the context as the features of the problem situation perceived only from the point of view of the researcher, i.e., most often as *external parameters of the problem*. Therefore, these features are considered to be superficial and irrelevant to the way the situation (problem) is understood by the students. The opposite approaches emphasize the context as highly individual and therefore necessarily mediated by students' previous knowledge and experiences, which, moreover, vary inter-individually. We consider this to be an important argument corresponding, among others, to our doubts about the real-life nature of the PISA problems.

On the other hand, even most of the critics overlook that the school learning – as we already stated above – generates a specific context tending to be similar for the students in one classroom. This happens, among others, because their previous knowledge and experiences consist, maybe even dominantly, in knowledge and experiences acquired during the teaching/learning of the given subject. The context of the school class serves as a support for inter-contextuality – or as a “focusing phenomenon” (Lobato, 2006), or “framing” (Engle, 2006) or “context sensitivity” (Wagner, 2006).

Then, the student logically gets into problems if he or she is suddenly deprived of this support. In our longitudinal research (Rendl, unpublished data) we observed this difficulty in the learning within the school itself: students manifested some difficulties in applying mathematics knowledge, even well acquired, to other school subjects (physics, geography, chemistry). It is evident that the leaving of the contextual support of the school context represents a specific problem. At the same time, it is questionable if it is the very school that can completely solve the problem of helping the students to find the meaning of school knowledge and of its effective use in everyday or even professional contexts which, moreover, significantly vary between individuals.

<sup>8</sup> “... abstraction was a *consequence* of transfer and the growth of understanding – not the cause of it” (ibid, p. 66).

To summarize the issue, the recent discussions about the transfer bring the following methodological and theoretical questions:

- Should we investigate the general conditions for improving the learning performance or explore the individual processes of qualitative changes during the learning process? Should we examine the consequences of learning in terms of the results of formalised activity within an experimental design or should we explore the process of learning that leads to the performance?
- Should we start from the “meaningless and rigid” abstraction moving to the activity in a concrete situation or start from the situated activity moving to the understood abstraction? From de-contextualization to contextualization or vice versa?
- Should we study the learning process as isolated from the surrounding context or should we analyze it as influenced by various context frames in their full scale – from classroom to family and to broader culture of student’s life?

We think that these contradictions comprehend real moments of the learning process and, in consequence, they contribute to shape the design of the research on learning. However, as we demonstrated, it is not reasonable to postulate them as antagonistic instead of complementary.

As the evidence-based discussion over the issues mentioned above demonstrates, it is also more and more clear that transfer and learning are one and the same process and that one of these terms seems to be redundant. In this, we are close to the opinion of Carraher and Schliemann (2002): “Our goal is to recommend not an ‘improved’ version of transfer, but rather the abandonment altogether of ‘transfer’ as a view of how learning takes place” (p. 22) “... there are other ways to frame the way prior knowledge and experience contribute to learning” (ibid).

## 4 Conclusion

The Czech reform documents and their interpreters relegate learning of knowledge and postulate competences as the main objective of learning. But ‘cognitive competences’ appears to be a very vague term. If the term is not used simply as a general term for knowledge, skills, abilities, etc., its extension remains unclear and is used very arbitrarily.

Our effort to operationalize competences using the data of TIMSS 2007 and PISA 2003 was not successful. In the TIMSS data, we were unable to identify didactic procedures (teaching classroom practices) that would distinguish “pro-reform” teachers (allegedly trying to develop the students’ competences) from those who are “anti-reform” (allegedly using inefficient traditional methods). Also, no differences in the mathematics test results were found in students taught by either of the above mentioned groups.

Similarly in PISA 2003 we identified no impact on the mathematics test results (said to measure the level of “competences for life”) which could be ascribed to didactic procedures – and thereby no differences in their effectivity could be found.

According to the reformists, the school policy emphasizing competences should also be realized by means of learning based on the RWP. Unfortunately, we did not find any discussion of competences in the field of psychological research. Instead of competences, the analyzed studies explore the use of RWP from the point of view of mathematical knowledge and of the ability to apply it. Moreover, the results of these studies are ambiguous. If the effectiveness of learning based on RWP is higher, it is mostly when the measurements are made on problems contextualized similarly to the original problems. On the other hand, some studies report that excessive use of RWP in teaching/learning may even have negative effects: the student remains fixed to a particular conceptualization connected with a particular contextualization and he/she fails to form more general concepts, categories and principles. The question, therefore, is not whether to use RWP or not, but when, in which phase of the learning of new items of knowledge, and in what way.

The transfer of knowledge acquired in school into everyday or even professional life appears to be another requirement the reformists expect the school to meet. But this element seems almost impossible to pin down in research. Recent discussions concerning the need for a new understanding of transfer raise the question of the usefulness of the term of transfer itself. As we are trying to argue, its use seems to create a parallel theory of learning, which, however, rests merely on a different nomenclature and not on a real difference in phenomena distinct from the phenomena of long-term learning.

We thus understand the discourse of Czech reformists as an extraction of partial moments of the teaching/learning process. After such an extraction, these partial moments are isolated as mutually incompatible general approaches to teaching/learning – the choice is either between direct transmission of knowledge by the teacher, to name but one such purported general approach, or, as its opposite, the construction of knowledge by students. The tendency to postulate different moments as antagonisms and their labelling as universally good or bad is also apparent in constructivism-inspired reforms of teaching mathematics in the USA and elsewhere as well as in research literature.

To put undue emphasis on one element in the real yet to no extent absolute dichotomy, or, indeed to portray that element as the only desirable type of approach, inevitably reduces the efficiency of teaching/learning. Decades of research, including recent enquiries, prove that there is no panacea, no teaching/learning method which would be effective under all circumstances. By the same token, however, we believe that there is no method of which one could under all circumstances say: “If it fails to help, well, at least it will not do harm.” All methods are capable of causing damage to the process of teaching/learning if they are applied without regard to the process as a whole, in excessive measure (one-sidedly) and without repeated consideration of both the cognitive level attained by the student and of the zone of his/her proximal development.

- Barnett, S. M., Ceci, S. J. (2002). When and where do we apply what we learn? A taxonomy for far transfer. *Psychological Bulletin*, 128, 612–637.
- Cole, M., & Scribner, S. (1981). *The psychology of literacy*. Cambridge MA: Harvard University Press.
- De Bono, E. (1976). *Teaching thinking*. Harmondsworth: Penguin.
- Engle, R. A. (2006). Framing interactions to foster generative learning: A situative explanation of transfer in a community of learners' classroom. *Journal of the Learning Sciences*, 15, 451–498.
- Ennis, R. H. (1987). A taxonomy of critical thinking dispositions and abilities. In *Teaching thinking skills: Theory and practice*. New York: W. H. Freeman and Company.
- Franklová, Z. (2009). Reforma SOŠ je, když... [Reforming the Professional High School]. *Rodina a škola*, 56, 22–23.
- Laval, Ch. (2004). *L'école n'est pas une entreprise. Le néo-libéralisme à l'assaut de l'enseignement public*. Paris: La Découverte.
- Lave, J. (1977). Tailor-made experiments and evaluation of the intellectual consequences of apprenticeship training. *Quarterly Newsletter of the Laboratory of Comparative Human Cognition*, 1, 1–3.
- Lave, J. (1988). *Cognition in practice*. Cambridge, UK: Cambridge University Press.
- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.
- Lobato, J. (2006). Alternative perspectives on the transfer of learning: History, issues, and challenges for future research. *The Journal of the Learning Sciences*, 15, 431–449.
- Learning mathematics for life: A perspective from PISA*. (2009). Paris: OECD.
- Maňák, J., Janík, T., & Švec, V. (2008). *Kurikulum v současné škole* [The Contemporary School Curriculum]. Brno: Paido.
- Mleziva, J.-M. (2011). Co s matematikou? [What about the Maths?] *Rodina a škola*, 58, s. 5.
- National Mathematics Advisory Panel (2008). *Final report*. U.S. Department of Education. Retrieved from <http://www2.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>.
- National Mathematics Advisory Panel (2008). *Report of the task group on learning processes*. U.S. Department of Education. Retrieved from <http://www2.ed.gov/about/bdscomm/list/mathpanel/report/learning-processes.pdf>.
- National Mathematics Advisory Panel (2008). *Report of the task group on instructional practices*. U.S. Department of Education. Retrieved from <http://www2.ed.gov/about/bdscomm/list/mathpanel/report/instructional-practices.pdf>.
- Paul, R. (1993). *Critical Thinking – What Every Person Needs to Survive in a Rapidly Changing World (3rd ed.)*. J. Willsen, & A. J. A. Binker (Eds.) Santa Rosa, CA: Foundation for Critical Thinking.
- Rámcový vzdělávací program pro základní vzdělávání* [Framework Education Programme for Basic Education]. (2005). Praha: VÚP.
- Rogoff, B. (1990). *Apprenticeship in thinking: Cognitive development in social context*. New York: Oxford University Press.
- Ropé, F., & Tanguy, L. (1994). *Savoirs et compétences. De l'usage de ces notions dans l'école et l'entreprise*. Paris: L'Harmattan.
- Scribner, S. (1986). Thinking in action: Some characteristics of practical thought. In R. J. Sternberg, & R. K. Wagner (Eds.) *Practical intelligence*. Cambridge, UK: Cambridge University Press, 13–30.
- Schliemann, A. D., & Carraher, D. W. (2002). The evolution of mathematical reasoning: Everyday versus idealized understandings. *Developmental Review*, 22, 242–266.
- Schliemann, A. D., & Carraher, D. W. (2002). The transfer dilemma. *The Journal of the Learning Sciences*, 11, 1–24.



- Son, J. I., & Goldstone, R. L. (2009). Contextualization in Perspective. *Cognition and Instruction*, 27, 51–89.
- Štech, S. (2008). School mathematics as a developmental activity. In A. Watson, & P. Winbourne, (Eds.) *New directions for situated cognition in mathematics education*. London: Springer, 13–30.
- Wagner, J. F. (2006). Transfer in pieces. *Cognition and Instruction*, 24, 1–71.

Miroslav Rendl  
Dept. of Psychology, Faculty of Education,  
Charles University in Prague,  
Myslíkova 7, 110 00 Prague,  
Czech Republic  
miroslav.rendl@pedf.cuni.cz

Stanislav Štech  
Dept. of Psychology, Faculty of Education,  
Charles University in Prague  
Myslíkova 7, 110 00 Prague,  
Czech Republic  
stanislav.stech@ruk.cuni.cz