MODERN METHODS OF CHARACTERIZATION OF MECHANICAL PROPERTIES OF VISCOELASTIC BODIES AND MECHANICAL MATCHING OF ARTIFICIAL AND BIOLOGICAL MATERIALS

STANISLAV ĎOUBAL, MONIKA KUCHAŘOVÁ, PETRA ŘEHÁČKOVÁ, ADÉLA HRADECKÁ

Department of Biophysics and Physical Chemistry, Faculty of Pharmacy in Hradec Králové,
Charles University in Prague, Czech Republic
e-mail: Doubal@faf.cuni.cz

ABSTRACT

This paper provides a survey of recent methodology of characterizing dynamic behavior of viscoelastic bodies. A special attention is paid to problems of mechanical compatibility of artificial and biological structures. A detailed analysis of mechanical matching of skin and plasters or bondages is presented. The criterion for mechanical matching of two membranes is derived.

Key words: viscoelasticity – complex modulus – mechanical compatibility – plasters – bondages

INTRODUCTION

This paper aims to provide a survey of recent methodology of characterizing dynamic behavior of viscoelastic bodies, including examples of its applications in pharmacy. Specific attention is paid to problems of mechanical compatibility of artificial and biological structures.

Applications of medical plasters, bandages, prothesis, replacements and many other artificial auxiliaries or supporting tools lead usually to permanent mechanical contact with biological structures. These artificial structures and adjoining biological materials should be mechanically compatible.[1] Mechanical compatibility means that the deformations are the same or, at least, sufficiently similar in both structures.

Mechanical incompatibility of these artificial structures and adjoining biological materials may lead to health problems and, in less critical situations, noticeable discomfort. The origin of these problems lies in tangent forces on the boundary between mechanically incompatible structures. These forces may result, among others, in tear-off plasters and irritation of adjacent biological structures. Generally, the mechanical incompatibility may lead to inflammation and other health complications. A disintegration of the connection between artificial and biological structures cannot be excluded.

In practice, the solution of these problems still depends mainly on experience. However, quantitative solution is principally possible, when modern theory of viscosity is applied and adequate methodology of measurements is used.[2]

THEORETICAL BACKGROUND

Solution of problems of mechanical matching requires knowledge of mechanical behavior of structures under study. Mechanical behavior may be defined as the relationship between forces and deformations. In case of viscoelastic bodies, including virtually all biological structures, mechanical behavior depends on theirs elastic as well as viscose properties.

Conventional approach to characterization of mechanical behavior of viscoelastic bodies is based on the identification of corresponding rheological model. Classical rheological models are structures combining several elastic and viscose bodies. The well known and often applied model is the Voigt's model. It is possible to demonstrate, theoretically as well as experimentally, that classical rheological models do not provide satisfactory characterization of dynamic behavior of viscoelastic bodies. Extended viscoelastic models with additional non-linear elements also do not lead to compliance with reality.

The more recent approach does not try to find a rheological model. Instead, the main idea is based on mathematical models and on theory of linear differential equations and Fourier transform.[1] Mechanical behavior of viscoelastic bodies is quantified by complex dynamic moduli or complex dynamic stiffnesses. These characteristics fully describe mechanical behavior of linear or linearized mechanical systems.

For complex stiffness $S(i\omega)$ it holds that:

$$S(i\omega) = \frac{F(i\omega)}{l(i\omega)} \tag{1}$$

where $F(i\omega)$ is the phasor representation of force, $l(i\omega)$ is the phasor representation of deformation, i is the imaginary unit $i = \sqrt{-1}$, ω is the angular frequency.

The knowledge of the relationships between amplitudes of forces and deformations is important in many situations. The ratio of the amplitude of force to the amplitude of deformation is given by the absolute value of the complex stiffness:

$$|S(\omega)| = \sqrt{S_{RE}^2(w) + S_{IM}^2(\omega)},$$

where S_{RE} is the real part of the complex stiffness and S_{IM} is the imaginary part of the complex stiffness.

Complex modulus is often used in the case of tensile loading. Analogically to the equation (1), the following equation holds for the complex modulus $E(i\omega)$:

$$E(i\omega) = \frac{\sigma(i\omega)}{\varepsilon(i\omega)} \tag{2}$$

where $\sigma(i\omega)$ is the phasor representation of stress, $\varepsilon(i\omega)$ is the phasor representation of strain.

The ratio of the amplitude of stress to the amplitude of strain is given by the absolute value of the complex dynamic modulus:

$$E(\omega) = \sqrt{E_{RE}^2(\omega) + E_{IM}^2(\omega)},$$

where E_{RE} is the real part of the complex modulus and E_{IM} is the imaginary part of the complex modulus.

The imaginary part of the complex modulus (loss modulus) characterizes dissipative energy losses in a mechanical system. The real part of the complex modulus (conservative or storage modulus) characterizes energy stored in a mechanical system.

Loss factor L is the ratio between loss and storage modulus.

$$L(i\omega) = \frac{E_{IM}}{E_{RE}}$$

The loss factor describes the ratio between dissipative energy and conservative energy in mechanical system during dynamic process.

In tensile loading of a uniform rod, the complex modulus may be calculated from its complex stiffness and the rod geometry according to the formula (3):

$$E(i\omega) = S(i\omega)\frac{l}{A} \tag{3}$$

where A is the cross section area of the rod, l is the length of the rod.

MEASUREMENT

Currently available apparatuses for measurement of complex dynamic moduli or complex dynamic stiffness's are so called Dynamic Mechanical Analyzers (DMA).[3] Measurements are based on the measurement of the relationship between harmonic forces and harmonic deformations. The DMA apparatuses measure the ratio between amplitudes of forces and amplitudes of deformations and the phase shift between forces and deformations for different frequencies. The results are frequency dependence of storage and loss moduli. Usually the loss factor is also calculated.

More modern measurements are based on mechanical resonance principle. Resonance Analyzers (RA) enable more sensitive measurements, RA are less complicated technically in comparison with DMA and less expensive.[4]

MECHANICAL MATCHING

Solution of problems of mechanical matching requires knowledge of the structure and the geometry of the system as well as knowledge of mechanical properties of components of the system. It may be a particularly complicated and difficult task. The analysis of mechanical matching of thin structures is relatively easy. Pharmaceutically interesting is for example the mechanical matching of skin and plasters, vessels walls and stents, or body surface and bandages.

For illustration, the analysis of mechanical matching of skin and plasters will be discussed in more details. Human skin as well as plasters may be considered as a membrane covering sub-dermal and other inner structures.[5] In membranes, differences of pressures inside and outside of the membrane lead to changes of the membrane geometry according to Laplace law.[6] For spherical shape of a membrane it holds:

$$T = \frac{PR}{2} \tag{4}$$

where *T* is the membrane tension, *P* is the pressure difference, *R* is the radius of curvature.

Membrane tension *T* is force per length (see Fig. 1):

$$T = \frac{F}{L}$$

For stress in membrane it holds:

$$\sigma = \frac{T}{d}$$

where *d* is the membrane thickness. Also holds:

$$\sigma = \frac{F}{dL}$$

and

$$\sigma = \frac{F}{A}$$

where *A* is the cross section area of membrane.

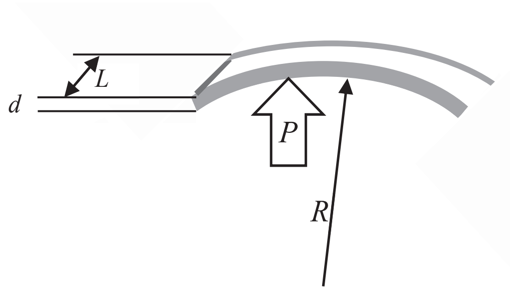

Fig 1. Illustration to Laplace law

Fig 2. Heterogeneous membrane and Laplace law

Relative deformation ε of membrane is (see formula 2):

$$\varepsilon(i\omega) = \frac{\sigma(i\omega)}{E(i\omega)}$$

Consequently, on the basis of measurement of pressure and radius of curvature, it is possible to calculate the membrane tension and the stress in the membrane. Furthermore, the deformation may also be calculated.

Membrane thickness is, according to its definition, much smaller than the curvature radius. Consequently tensions in membranes are the same (see Fig. 2). For strains in membranes holds:

$$\sigma_1 = \frac{T}{d_1}$$

and

$$\sigma_2 = \frac{T}{d_2}$$

As the condition for mechanical matching is

$$\varepsilon_1 = \varepsilon_2,$$

the condition of mechanical matching for two membranes is:

$$\frac{E_1(i\omega)}{E_2(i\omega)} = \frac{d_2}{d_1}$$

Example: Elastic modulus of material of medical plaster is 10 MPa. Suppose that the plaster is applied on skin. Suppose that elastic modulus of skin is 1 MPa and thickness of skin is 1 mm. Mechanical matching thus requires the thickness of plaster 0.1 mm.

The analysis outlined above illustrates basic principles of suggested methodology. It is limited to simple model situation. Naturally, the general solution of problems connected with mechanical matching of real heterogeneous structures requires more detail analysis.

CONCLUSIONS

Albeit the solution of mechanical matching of heterogeneous structures is in general a complicated problem, application of Laplace law may lead to relatively easy methodology in case of mechanical matching of walls surrounding large objects. The Young–Laplace equation (Laplace law) relates the pressure difference to the shape changes and wall tension (eq. 4) in case of a membrane surrounding large objects. In other words, radii of curvature must be much greater than the thickness of walls.

The methodology outlined in this paper may be applied to solve problems of mechanical matching of medical plasters and bandages. We hope that it will help manufactures to improve quality of these products. On the other hand, application of this theory requires appropriate experimental equipment.[7,8] Mainly meters for determination of strain-stress diagrams and complex moduli are necessary.

References

1. Ďoubal, S., Klemera, P., Kuchařová, M.: Mechanical Behavior of Viscoelastic bodies (in Czech), Prague, Karolinum, 2011.
2. Kuchařová, M., Ďoubal, S., Klemera, P., Rejchrt, P., Navrátil, M.: Viscoelasticity of biological materials – measurement and practical impact on biomedicine. Physiol. Res. (Suppl. 1), 56, 2007, 33–37.
3. Lakes, R. S.: Viscoelastic measurement techniques. Review of Scientific Instruments, 75, 2004, 797–810.
4. Ďoubal, S., et al.: Appliance for measurement of viscoelastic materials. Charles University in Prague, 2010, Utility. 20792, 9.4.
5. Ďoubal, S., Klemera, P.: Visco-elastic response of human skin and aging. J. Amer. Aging Assoc., 3, 2002, 115–117.
6. Basford, J. R.: The law of Laplace and its relevance to contemporary medicine and rehabilitation. Arch Phys Med Rehabil., 2002, 83, 1165–70.
7. Kuchařová, M., Ďoubal, S., Klemera, P., Rejchrt, P., Navrátil, M.: Viscoelasticity of biological materials – measurement and practical impact on biomedicine. Physiol. Res. (Suppl. 1), 56, 2007, 33–37.
8. Ďoubal, S., et al.: A New Approach to Measurement of Viscoelastic Parameters of Biological Structures. Proceedings of Second Fascia Research Congress, 2009, Amsterdam, Vrije Universiteit, 107.